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ON THE VALUES OF INDEPENDENCE AND DOMINATION POLYNOMIALS AT SPECIFIC POINTS

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ABSTRACT. Let G be a simple graph of order n . We consider the independence polynomial and the domination polynomial of a graph G . The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. In this paper we investigate independence and domination polynomial at -1 and 1 .

1. Introduction

Let $G = (V, E)$ be a simple graph. For any vertex $v \in V$, the *open neighborhood* of v is the set $N(v) = \{u \in V | uv \in E\}$ and the *closed neighborhood* is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. The *corona* of two graphs G_1 and G_2 , as defined by Frucht and Harary in [12], is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 , where the i th vertex of G_1 is adjacent to every vertex in the i th copy of G_2 . The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G , where for each vertex $v \in V(G)$, a new vertex v' and a pendant edge vv' are added. The *join* of two graphs G_1 and G_2 , denoted by $G_1 + G_2$ is a graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv | u \in V(G_1) \text{ and } v \in V(G_2)\}$.

Graph polynomials are a well-developed area useful for analyzing properties of graphs. The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. Balister, et al. in [9] proved that for any graph G , $|q(G, -1)|$ is always a power

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of 2, where $q(G, x)$ is interlace polynomial of a graph G . Stanley in [14] proved that $(-1)^n P(G, -1)$ is the number of acyclic orientations of G , where $P(G, \lambda)$ is the chromatic polynomial of G and $n = |V(G)|$.

In this paper we study independence and domination polynomial of a graph at -1 and 1 . For convenience, the definition of the independence and domination polynomial of a graph will be given in the next sections.

We denote the path of order n , the cycle of order n , and the wheel of order n , by P_n , C_n , and W_n , respectively.

2. THE INDEPENDENCE POLYNOMIAL OF A GRAPH AT -1 AND 1

An *independent set* of a graph G is a set of vertices where no two vertices are adjacent. The *independence number* is the size of a maximum independent set in the graph and denoted by $\alpha(G)$. For a graph G , let i_k denote the number of independent sets of cardinality k in G ($k = 0, 1, \dots, \alpha$). The *independence polynomial* of G ,

$$I(G, x) = \sum_{k=0}^{\alpha} i_k x^k,$$

is the generating polynomial for the independent sequence $(i_0, i_1, i_2, \dots, i_\alpha)$.

$I(G; 1) = i_0 + i_1 + i_2 + \dots + i_\alpha$ equals the number of independent sets of G , and $I(G; -1) = i_0 - i_1 + i_2 - \dots + (-1)^\alpha i_\alpha = f_0(G) - f_1(G)$, where $f_0(G) = i_0 + i_2 + i_4 + \dots$, $f_1(G) = i_1 + i_3 + i_5 + \dots$ are equal to the numbers of independent sets of even size and odd size of G , respectively. $I(G; -1)$ is known as the alternating number of independent sets.

We need the following theorem:

Theorem 2.1. ([13]) *If $w \in V(G)$ and $uv \in E(G)$, then the following equalities hold:*

- (i) $I(G; x) = I(G - w; x) + xI(G - N[w]; x)$,
- (ii) $I(G; x) = I(G - uv; x) - x^2 I(G - N(u) \cup N(v); x)$.

A vertex v is pendant if its neighborhood contains only one vertex; an edge $e = uv$ is pendant if one of its endpoints is a pendant vertex.

Theorem 2.2. *If $u \in V(G)$ is a pendant vertex of G and $v \in N(u)$, then*

$$I(G; -1) = -I(G - N[v]; -1).$$

Proof. Since $u \in V(G)$ is a pendant vertex of G and $v \in N(u)$, Theorem 2.1 assures that $I(G; x) = I(G - v; x) + xI(G - N[v]; x) = (1 + x)I(G - \{u, v\}; x) + xI(G - N[v]; x)$. Therefore $I(G; -1) = -I(G - N[v]; -1)$. \square

Theorem 2.3. For $n \geq 1$, the following hold:

- (i) $I(P_{3n-2}, -1) = 0$ and $I(P_{3n-1}, -1) = I(P_{3n}, -1) = (-1)^n$;
- (ii) $I(C_{3n}, -1) = 2(-1)^n$, $I(C_{3n+1}, -1) = (-1)^n$ and $I(C_{3n+2}, -1) = (-1)^{n+1}$;
- (iii) $I(W_{3n+1}, -1) = 2(-1)^n - 1$ and $I(W_{3n}, -1) = I(W_{3n+2}, -1) = (-1)^n - 1$.

Proof.

- (i) We prove by induction on n . For $n = 1$, since $I(P_1, x) = 1 + x$, $I(P_2, x) = 1 + 2x$ and $I(P_3, x) = 1 + 3x + x^2$, we have $I(P_1, -1) = 0$ and $I(P_2, -1) = I(P_3, -1) = -1$. Suppose that the result is true for any $k \leq 3n$. By Theorem 2.1(i), $I(P_{k+1}, x) = I(P_k, x) + xI(P_{k-1}, x)$, therefore

$$\begin{aligned} I(P_{3n+1}, -1) &= I(P_{3n}, -1) - I(P_{3n-1}, -1) = (-1)^n - (-1)^n = 0; \\ I(P_{3n+2}, -1) &= I(P_{3n+1}, -1) - I(P_{3n}, -1) = 0 - (-1)^n = (-1)^{n+1}; \\ I(P_{3n+3}, -1) &= I(P_{3n+2}, -1) - I(P_{3n+1}, -1) = (-1)^{n+1} - 0 = (-1)^{n+1} \end{aligned}$$

- (ii) Since $I(C_3, x) = 1 + 3x$, $I(C_4, x) = 1 + 4x + 2x^2$, $I(C_5, x) = 1 + 5x + 5x^2$ and $I(C_6, x) = 1 + 6x + 9x^2 + 2x^3$, we have $I(C_3, -1) = 2(-1)$, $I(C_4, -1) = -1$ and $I(C_5, -1) = (-1)^2$. By Theorem 2.1 and Part (i), $I(C_n, x) = I(P_{n-1}, x) + xI(P_{n-3}, x)$. Therefore $I(C_n, -1) = I(P_{n-1}, -1) - I(P_{n-3}, -1)$. Now it is easy to see that $I(C_{3k}, -1) = 2(-1)^k$, $I(C_{3k+1}, -1) = (-1)^k$ and $I(C_{3k+2}, -1) = (-1)^{k+1}$.
- (iii) By Theorem 2.1(i), $I(W_n, x) = I(C_{n-1}, x) + x$. Now by part (ii) we have the result. \square

In the following theorem we compute the number of independent sets of paths, cycles and wheels:

Theorem 2.4. (i) For any positive integer n ,

$$I(P_n, 1) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(1 + 4 \cos^2 \frac{s\pi}{n+2} \right).$$

- (ii) For any integer $n \geq 3$,

$$I(C_n, 1) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(1 + 4 \cos^2 \frac{(s - \frac{1}{2})\pi}{n} \right).$$

- (iii) For any integer $n \geq 4$,

$$I(W_n, 1) = 1 + \prod_{s=1}^{\lfloor \frac{n-1}{2} \rfloor} \left(1 + 4 \cos^2 \frac{(s - \frac{1}{2})\pi}{n-1} \right).$$

Proof. We know that (see [8]), $I(P_n, x) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(2x + 1 + 2x \cos \frac{2s\pi}{n+2} \right)$, $I(C_n, x) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(2x + 1 + 2x \cos \frac{(2s-1)\pi}{n} \right)$ and $I(W_n, x) = I(C_{n-1}, x) + x$. By substituting $x = 1$, we have the results. \square

We recall the following theorem:

Theorem 2.5. ([3, 11]) *If G has t connected components G_1, \dots, G_t , then $I(G, x) = \prod_{i=1}^t I(G_i, x)$.*

Theorem 2.6. *For any tree T , $I(T; -1) \in \{-1, 0, 1\}$, i.e., the number of independent sets of even size varies by at most one from the number of independent sets of odd size.*

Proof. By induction on $n = |V(T)|$. It is not hard to see that for any tree with order 1, 2, 3, 4, 5 we have the result. Now suppose that $|V(T)| = n + 1 \geq 5$, v is a pendant vertex of T , $N(v) = \{u\}$ and T_1, T_2, \dots, T_k are the trees of the forest $T - N[u]$. According to Theorem 2.1(ii), we have:

$$\begin{aligned} I(T; x) &= I(T - uv; x) - x^2 I(T - N(u) \cup N(v); x) = \\ &= (1 + x)I(T - v; x) - x^2 I(T - N[u]; x) = \\ &= (1 + x)I(T - v; x) - x^2 I(T_1; x)I(T_2; x) \dots I(T_k; x) \end{aligned}$$

Therefore $I(T; -1) = -I(T_1; -1)I(T_2; -1) \dots I(T_k; -1) \in \{-1, 0, 1\}$, since every tree T_i has less than n vertices, by the induction hypothesis, $I(T_i; -1) \in \{-1, 0, 1\}$. \square

Theorem 2.7. *If F is a forest, then $I(F; -1) \in \{-1, 0, 1\}$.*

Proof. If T_1, T_2, \dots, T_k , $k \geq 1$, are the connected components of F , then $I(F; x) = I(T_1; x)I(T_2; x) \dots I(T_k; x)$, because $F = T_1 \cup T_2 \cup \dots \cup T_k$. Now, the conclusions follow from Theorem 2.6. \square

Theorem 2.8. ([3, 11]) $I(G_1 + G_2; x) = I(G_1; x) + I(G_2; x) - 1$.

Theorem 2.9. *For any integer k there is some connected graph G such that $I(G; -1) = k$.*

Proof. First suppose that k is a negative integer. It is easy to check that $I(K_{a,a,\dots,a}, x) = n(1+x)^a - (a-1)$, and therefore $I(K_{a,a,\dots,a}, -1) = 1 - a$ i.e., for any negative integer k there is some connected graph G such that $I(G; -1) = k$. Now suppose that k is a positive integer. Let G_i , $1 \leq i \leq k$, be k graphs with $I(G_i; -1) = 2$ for every $i \in \{1, 2, \dots, k\}$ (for example we can put $G_i = C_6$ for every $i \in \{1, 2, \dots, k\}$) and $H = G_1 + G_2 + \dots + G_k$. Then

$$I(H, x) = I(G_1, x) + I(G_2, x) + \dots + I(G_k, x) - (k - 1)$$

and consequently $I(H, -1) = k + 1$. The case of $k = 0$ follows from Theorem 2.6. \square

3. THE DOMINATION POLYNOMIAL AT -1 AND 1

A set $S \subseteq V$ is a *dominating set* if $N[S] = V$, or equivalently, every vertex in $V \setminus S$ is adjacent to at least one vertex in S . An *i -subset* of $V(G)$ is a subset of $V(G)$ of cardinality i . Let $\mathcal{D}(G, i)$ be the family of dominating sets of G which are i -subsets and let $d(G, i) = |\mathcal{D}(G, i)|$. The polynomial $D(G, x) = \sum_{i=1}^{|V(G)|} d(G, i)x^i$ is defined as *domination polynomial* of G ([1, 4, 5]). Every root of $D(G, x)$ is called a *domination root* of G . The set of all roots of $D(G, x)$ is denoted by $Z(D(G, x))$. Let $A \subseteq B \subseteq V$. Define $\mathcal{D}_{A,B}(G, i)$ as follows

$$\mathcal{D}_{A,B}(G, i) = \{S \in \mathcal{D}(G, i) \mid S \cap B = A\}.$$

Let $d_{A,B}(G, i) = |\mathcal{D}_{A,B}(G, i)|$ and define $D_{A,B}(G, x) = \sum_{i=1}^{|V|} d_{A,B}(G, i)x^i$.

We shall consider the value of domination polynomial at -1 and 1.

Note that $D(G; 1) = d(G, 1) + d(G, 2) + \dots + d(G, n)$ equals the number of dominating sets of G , and $D(G; -1) = -d(G, 1) + d(G, 2) - d(G, 3) + \dots + (-1)^n d(G, n) = g_0(G) - g_1(G)$, where $g_0(G) = d(G, 2) + d(G, 4) + d(G, 6) + \dots$, $g_1(G) = d(G, 1) + d(G, 3) + d(G, 5) + \dots$ are equal to the numbers of dominating sets of even size and odd size of G , respectively. We call $D(G; -1)$ as the alternating number of dominating sets.

The following theorem gives the value of domination polynomial of complete graphs, complete bipartite graphs and stars at 1 and -1 :

Theorem 3.1. (i) $D(K_n, -1) = 1$, $D(K_{m,n}, -1) \in \{\pm 1, 3\}$ and $D(K_{1,n}, -1) \in \{-1, 1\}$.
(ii) $D(K_n, 1) = D(K_{1,n}, 1) = 2^n + 1$ and $D(K_{m,n}, 1) = (2^m - 1)(2^n - 1) + 2$.

Proof. The results follow easily from $D(K_n, x) = (1+x)^n + 1$ and $D(K_{m,n}, x) = ((1+x)^m - 1)((1+x)^n - 1) + x^m + x^n$ (see [4]). \square

Theorem 3.2. Let G be a graph. If G has a vertex v of degree k such that $N[v]$ is a clique, then

$$D(G, x) = xD(G - N[v], x) + (1+x) \sum_{\emptyset \neq A \subseteq N(v)} D_{A, N(v)}(G - v, x).$$

where N_2, \dots, N_{2^k} are all nonempty subsets of $N(v)$. Moreover, $D(G, -1) = -D(G - N[v], -1)$.

Proof. Suppose that $N(v) = \{v_1, \dots, v_k\}$. Let $N_1 = \emptyset, N_2, \dots, N_{2^k}$ be all subsets of $N(v)$. Let A be a dominating set of G with cardinality i . We have the following cases:

- 1) $v \in A$ and $A \cap N(v) = N_1$, then $A - \{v\}$ is a dominating set of $G - N[v]$ with cardinality $i - 1$.
- 2) $v \in A$ and there exists an index t , $2 \leq t \leq 2^k$ such that $A \cap N(v) = N_t$. Since $N[v]$ is a clique, $A - \{v\}$ is a dominating set of $G - v$ with cardinality $i - 1$ and $(A - \{v\}) \cap N(v) = N_t$.
- 3) $v \notin A$. Therefore $A \cap N(v) \neq \emptyset$ and there exists an index l , $2 \leq l \leq 2^k$ such that $A \cap N(v) = N_l$. Therefore A is a dominating set of $G - v$ with cardinality i and $A \cap N(v) = N_l$.

By considering the above cases we obtain that

$$d(G, i) = d(G - N[v], i - 1) + \sum_{r=2}^{2^k} d_{N_r, N(v)}(G - v, i - 1) + \sum_{r=2}^{2^k} d_{N_r, N(v)}(G - v, i).$$

Therefore we have

$$D(G, x) = xD(G - N[v], x) + (1 + x) \sum_{r=2}^{2^k} D_{N_r, N(v)}(G - v, x).$$

Now, by putting $x = -1$ in the above equality the proof is complete. \square

As a consequence of the previous theorem we determine the exact value of the domination polynomial of a forest at -1 . We recall the following theorem:

Theorem 3.3. ([4]) *If G has t connected components G_1, \dots, G_t , then $D(G, x) = \prod_{i=1}^t D(G_i, x)$.*

Theorem 3.4. *Let F be a forest. Then $D(F, -1) = (-1)^{\alpha(F)}$.*

Proof. By Theorem 3.3 it is sufficient to prove the theorem for trees. We prove this theorem by induction on n , where n is the order of tree T . For $n = 1, 2$ there is nothing to prove. Now, Let $n \geq 3$. Let u_0 be an arbitrary vertex of T . Assume that w is a vertex with maximum distance of u_0 . Clearly $d(w)=1$. Let $vw \in E(T)$. Obviously, at most one of the neighbors of v has degree more than one. Since $n \geq 3$, then $d(v) \geq 2$. Assume that $d(v) = t$ and $\{u_1, \dots, u_{t-1}\} \subseteq N(v)$ and $d(u_i)=1$, for $i = 1, \dots, t - 1$. Let $H = T - \{v, u_1, \dots, u_{t-1}\}$. Clearly, H is a tree and $\alpha(T) = \alpha(H) + t - 1$. Now, By Theorems 3.3 and 3.2 and induction hypothesis we have

$$D(T, -1) = -D(T - N[u_1], -1) = -D(H, -1)(-1)^{t-2} = (-1)^{\alpha(H)}(-1)^{t-1}.$$

Therefore $D(T, -1) = (-1)^{\alpha(T)}$ and the proof is complete. \square

By Theorem 3.4 we have the following corollary:

Corollary 3.5. *For any forest F , $D(F; -1) \in \{-1, 1\}$, i.e., the number of dominating sets of even size varies by at most one from the number of dominating sets of odd size.*

Lemma 3.6. ([2]) *If n is a positive integer, then*

$$D(C_n, -1) = \begin{cases} 3 & \text{if } n \equiv 0 \pmod{4}; \\ -1 & \text{otherwise.} \end{cases}$$

Corollary 3.7. *For any $n \geq 4$, $D(W_n, -1) \in \{-1, 3\}$.*

Proof. Since for every $n \geq 4$, $D(W_n, x) = x(1+x)^{n-1} + D(C_{n-1}, x)$ ([4]), the result follows from Lemma 3.6. \square

Since $D(P_n, x) = x[D(P_{n-1}, x) + D(P_{n-2}, x) + D(P_{n-3}, x)]$ and $D(C_n, x) = x[D(C_{n-1}, x) + D(C_{n-2}, x) + D(C_{n-3}, x)]$ (see [6, 7]) we have the following theorem:

- Theorem 3.8.** (i) For every $n \geq 4$, $D(P_n, 1) = D(P_{n-1}, 1) + D(P_{n-2}, 1) + D(P_{n-3}, 1)$ with initial values $D(P_1, 1) = 1, D(P_2, 1) = 3$ and $D(P_3, 1) = 5$.
- (ii) For every $n \geq 4$, $D(C_n, 1) = D(C_{n-1}, 1) + D(C_{n-2}, 1) + D(C_{n-3}, 1)$ with initial values $D(C_1, 1) = 1, D(C_2, 1) = 3$ and $D(C_3, 1) = 7$.
- (iii) For any $n \geq 4$, $D(W_n, 1) = D(C_{n-1}, 1) + 2^{n-1}$.

Theorem 3.9. Let G be a unicycle graph. Then $D(G, -1) \in \{\pm 1, \pm 3\}$.

Proof. We prove the theorem by induction on $n = |V(G)|$. If $n = 3$, then $G = K_3$ and there is nothing to prove. Now, let $n \geq 4$. If $G = C_n$, then by Lemma 3.6, $D(G, -1) \in \{-1, 3\}$. If $G \neq C_n$, then there exists a vertex u such that $d(u) = 1$. Suppose that $N(u) = \{v\}$. Clearly, at most one of the components of $G - N[u]$ is a unicycle graph. Now, by Theorems 3.3, 3.2 and 3.4 and induction hypothesis the proof is complete. \square

Theorem 3.10. ([10]) For every graph, the number of dominating sets is odd.

Theorem 3.11. Let G be a graph. Then $D(G, r)$ is odd for every odd integer r . In particular $D(G, -1)$ is odd.

Proof. Suppose that r, s are two odd integers. Clearly, for every graph G , $D(G, r) \equiv D(G, s) \pmod{2}$. Therefore $D(G, r) \equiv D(G, 1) \pmod{2}$. So by Theorem 3.10, we conclude $D(G, r)$ is odd. Therefore $D(G, -1)$ is odd. \square

The following corollary is an immediate consequence of Theorem 3.11.

Corollary 3.12. Every integer domination root of a graph G is even.

We need the following theorem to prove another results:

Theorem 3.13. ([4]) Let G_1 and G_2 be graphs of orders n_1 and n_2 , respectively. Then

$$D(G_1 + G_2, x) = \left((1+x)^{n_1} - 1 \right) \left((1+x)^{n_2} - 1 \right) + D(G_1, x) + D(G_2, x).$$

The following corollary is an immediate consequence of above theorem:

Corollary 3.14. *For any two graphs G_1 and G_2*

$$D(G_1 + G_2, -1) = D(G_1, -1) + D(G_2, -1) + 1.$$

Theorem 3.15. *For any odd integer $n = 2k - 1$ ($k \in \mathbb{Z}$), there is some connected graph G such that $D(G; -1) = n$.*

Proof. Let G_i ($1 \leq i \leq k$) be k graphs with $D(G_i, -1) = 1$ for every $i \in \{1, 2, \dots, k\}$ and $H = G_1 + G_2 + \dots + G_k$. Then

$$D(H, x) = D(G_1, x) + \dots + D(G_k, x) + (k - 1)$$

and consequently, $D(H, -1) = k + (k - 1) = 2k - 1 = n$. So the result is true for positive odd integer. Now we consider negative odd integer. Let G_i ($1 \leq i \leq k$) be k graphs with $D(G_i, -1) = -3$ for every $i \in \{1, 2, \dots, k\}$ and $H = G_1 + G_2 + \dots + G_k$. Then

$$D(H, x) = D(G_1, x) + \dots + D(G_k, x) + (k - 1)$$

and consequently, $D(H, -1) = -3k + (k - 1) = -2k - 1$. \square

As an example for positive odd integer (in above theorem) we consider complete n -partite graphs. It is easy to check that for the complete n -partite graph K_{m_1, m_2, \dots, m_n} we have

$$D(K_{m_1, m_2, \dots, m_n}, x) = \sum_{i=m_1}^{m_n} x^i + \sum_{i=2}^n ((1+x)^{m_i} - 1)((1+x)^{m_1+\dots+m_{i-1}} - 1).$$

Suppose that for every $1 \leq i \leq n$ the number m_i is even. By above formula we have

$$D(K_{m_1, m_2, \dots, m_n}, -1) = \sum_{i=m_1}^{m_n} (-1)^i + \sum_2^n 1$$

Since all of m_i are even, we have $D(K_{m_1, \dots, m_n}, -1) = 2n - 1$.

Now we give an example for negative odd integer (in above theorem). Suppose that G is a unicycle graph G with $D(G, -1) = -3$ (see Theorem 3.9). Consider $H = \underbrace{G + G + \dots + G}_{n\text{-times}}$. It is easy to see that $D(H, -1) = -2k + 1$.

In this paper we studied independence and domination polynomial at -1 and 1 . There are many graphs which have -2 as domination roots (as an example all graphs of the form $H \circ K_1$ have this property, see [1]). We think that the study of the value of domination polynomial of a graph at -2 is important, because we have the following conjecture:

Conjecture 3.16. *If r is an integer domination root of a graph, then $r = 0$ or $r = -2$.*

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REFERENCES

- [1] S. Akbari, S. Alikhani, and Y. H. Peng, Characterization of graphs using domination polynomial, *European J. Combin.*, **31** (2010) 1714-1724.
- [2] S. Akbari and M. R. Oboudi, Cycles are determined by their domination polynomials, *Ars Combin.*, to appear.
- [3] Y. Alavi, P. J. Malde, A. J. Schwenk and P. Erdős, The vertex independence sequence of a graph is not constrained, *Congr. Numer.*, **58** (1987) 15-23.
- [4] S. Alikhani and Y. H. Peng, Introduction to domination polynomial of a graph, *Ars Combin.*, in press. Available at <http://arxiv.org/abs/0905.2251>.
- [5] S. Alikhani, Y. H. Peng, Domination polynomials of cubic graphs of order 10, *Turkish J. Math.*, **35** (2011) 355-366.
- [6] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomial of paths, *Int. J. Math. Math. Sci.*, **2009**, Article ID 542040, 10 pages doi:10.1155/2009/542040.
- [7] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomials of certain graphs, II, *Opuscula Math.*, **30** No. 1 (2010) 37-51.
- [8] S. Alikhani and Y. H. Peng, Independence roots and independence fractals of certain graphs, *J. Appl. Math. Comput.*, **36** (2011) 89-100.
- [9] P. N. Balister, B. Bollobas, J. Cutler, and L. Pebody, The interlace polynomial of graphs at -1 , *European J. Combin.*, **23** (2002) 761-767.
- [10] A. E. Brouwer, The number of dominating sets of a finite graph is odd, preprint.
- [11] J. I. Brown, K. Dilcher and R. J. Nowakowski, Roots of independence polynomials of well covered graphs, *J. Algebraic Combin.*, **11** (2000) 197-210.
- [12] R. Frucht and F. Harary, On the corona of two graphs, *Aequationes Math.*, **4** (1970) 322-325.
- [13] I. Gutman and F. Harary, Generalizations of the matching polynomial, *Utilitas Math.*, **24** (1983) 97-106.
- [14] R. P. Stanley, Acyclic orientations of graphs, *Discrete Math.*, **5** (1973) 171-178.

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