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THE CONDITION FOR A SEQUENCE TO BE POTENTIALLY $A_{L,M}$- GRAPHIC

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Abstract. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. The complete product split graph on $L+M$ vertices is denoted by $\overline{S}_{L,M} = K_L \vee \overline{K}_M$, where K_L and K_M are complete graphs respectively on $L = \sum_{i=1}^{p} r_i$ and $M = \sum_{i=1}^{p} s_i$ vertices with r_i and s_i being integers. Another split graph is denoted by $S_{L,M} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} \vee \cdots \vee \overline{S}_{r_p,s_p} = (K_{r_1} \vee \overline{K}_{s_1}) \vee (K_{r_2} \vee \overline{K}_{s_2}) \vee \cdots \vee (K_{r_p} \vee \overline{K}_{s_p})$. A sequence $\pi = (d_1, d_2, \ldots, d_n)$ is said to be potentially $S_{L,M}$-graphic (respectively $\overline{S}_{L,M}$)-graphic if there is a realization G of π containing $S_{L,M}$ (respectively $\overline{S}_{L,M}$) as a subgraph. If π has a realization G containing $S_{L,M}$ on those vertices having degrees $d_1, d_2, \ldots, d_{L+M}$, then π is potentially $A_{L,M}$-graphic. A non-increasing sequence of non-negative integers $\pi = (d_1, d_2, \ldots, d_n)$ is potentially $A_{L,M}$-graphic if and only if it is potentially $S_{L,M}$-graphic. In this paper, we obtain the sufficient condition for a graphic sequence to be potentially $A_{L,M}$-graphic and this result is a generalization of that given by J. H. Yin on split graphs.

1. Introduction

Let $G(V, E)$ be a simple graph (a graph without multiple edges and loops) with n vertices and m edges having vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. There are several famous results, Havel and Hakimi [3, 4] and Erdös and Gallai [1] which give necessary and sufficient conditions for a sequence

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$\pi = (d_1, d_2, \ldots, d_n)$ to be the degree sequence of a simple graph G. A combinatorial characterization of degree sequences can be seen in [6]. The disjoint union of the graphs G_1 and G_2 is written as $G_1 \cup G_2$. If $G_1 = G_2 = G$, we write $G_1 \cup G_2$ as $2G$. Further K_k, C_k, T_k and P_k respectively denote a complete graph on k vertices, a cycle on k vertices, a tree on $k+1$ vertices and a path on $k+1$ vertices. A clique is a maximal complete subgraph. If $\pi = (d_1, d_2, \ldots, d_n)$ is a graphic sequence, we denote by $\sigma(\pi) = d_1 + d_2 + \cdots + d_n$. A graphic sequence π is potentially H-graphic if there is a realization of π containing H as a subgraph, while π is forcibly H graphic if every realization of π contains H as a subgraph. If π has a realization in which the r vertices of largest degree induce a clique, then π is said to be potentially A_r-graphic. It is shown in [2] that if π is a graphic sequence with a realization G containing H as a subgraph, then there is a realization G of π containing H with the vertices of H having $|V(H)|$ largest degree of π. We know that a graphic sequence π is potentially K_{k+1}-graphic if and only if π is potentially A_{k+1}-graphic [10].

In this paper, we obtain the sufficient condition for a graphic sequence to be potentially $A_{L,M}$-graphic and this result is a generalization of that given by Yin [11] on split graphs. The main results of this paper are Theorems 3.3 and 3.4.

2. **Definitions and preliminaries**

We have the following definitions.

Definition 2.1. *The join (complete product) of two graphs G_1 and G_2 is a graph $G = G_1 \vee G_2$ with vertex set $V(G_1) \cup V(G_2)$ and the edge set consisting of all the edges of G_1 and G_2 together with the edges joining each vertex of G_1 with every vertex of G_2.*

Definition 2.2. [9]. *The split graph of the complete graphs K_r and K_s, denoted by $\overline{S}_{r,s}$, is the graph $K_r \vee \overline{K}_s$ having $r+s$ vertices, where $\overline{K}_s$ is the complement of K_s. We note that $\overline{S}_{r,1} = K_{r+1}$. For example, let $r = 3$ and $s = 4$, we form the split graph $\overline{S}_{3,4} = K_3 \vee \overline{K}_4$. We take $V(K_3) = \{v_1, v_2, v_3\}$ and $V(\overline{K}_4) = \{u_1, u_2, u_3, u_4\}$. The resulting split graph $\overline{S}_{3,4}$ on seven vertices is shown in Figure 1.*

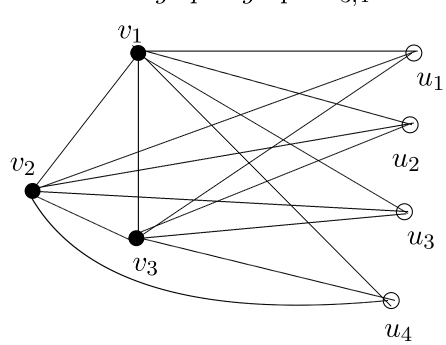

Figure 1 ($\overline{S}_{3,4}$)

Let $r_1, r_2, \ldots, r_p$ and $s_1, s_2, \ldots, s_p$ be positive integers and let $\sum_{i=1}^{p} r_i = L$ and $\sum_{i=1}^{p} s_i = M$. The split graph (by Definition 2.1) in this case is denoted by $\overline{S}_{L,M} = K_L \vee \overline{K}_M$.

Definition 2.3. *Let $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$ be split graphs, respectively with r_1+s_1, r_2+s_2,..,r_p+s_p vertices. The split graph in this case, denoted by $S_{L,M}$, is the complete product of $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$. We write*

$$\begin{aligned} S_{L,M} &= \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} \vee \cdots \vee \overline{S}_{r_p,s_p} \\ &= (K_{r_1} \vee \overline{K}_{s_1}) \vee (K_{r_2} \vee \overline{K}_{s_2}) \vee \cdots \vee (K_{r_p} \vee \overline{K}_{s_p}). \end{aligned}$$

Clearly $S_{L,M}$ has vertex set $\bigcup_{i=1}^{p} V(\overline{S}_{r_i,s_i})$ and the edge set consists of all the edges of $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$ together with the edges joining each vertex of $\overline{S}_{r_i,s_i}$ with every vertex of $\overline{S}_{r_j,s_j}$ for every i, j with $i \neq j$.

Remark 2.4. *$\overline{S}_{L,M}$ is always a subgraph of $S_{L,M}$.*

For example, let $r_1 = 2$, $r_2 = 2$, $s_1 = 1$ and $s_2 = 2$. Take $L = r_1 + r_2 = 2 + 2 = 4$, and $M = s_1 + s_2 = 1 + 2 = 3$. We construct the split graphs $\overline{S}_{L,M}$ and $S_{L,M}$ as follows.

We have, $\overline{S}_{L,M} = \overline{S}_{3,4} = K_3 \vee \overline{K_4}$. This is shown in Figure 1.

We have $\overline{S}_{r_1,s_1} = \overline{S}_{2,1} = K_2 \vee \overline{K_1}$ and $\overline{S}_{r_2,s_2} = \overline{S}_{2,2} = K_2 \vee \overline{K_2}$. We form the complete product split graph $S_{L,M} = S_{3,4} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} = \Big(K_2 \vee \overline{K_1}\Big) \vee \Big(K_2 \vee \overline{K_2}\Big)$. We take the vertex set of $\overline{S}_{r_1,s_1} = \overline{S}_{2,1}$ as $\{u_1, u_2, v_1\}$, and the vertex set of $\overline{S}_{r_2,s_2} = \overline{S}_{2,2}$ as $\{w_1, w_2, x_1, x_2\}$. The resulting complete product split graph $S_{L,M} = S_{3,4}$ on seven vertices is shown in Figure 3.

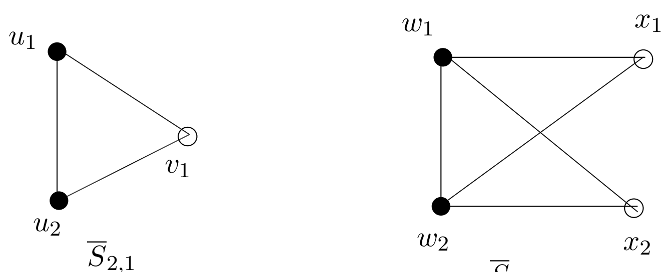

Figure 2

A sequence $\pi = (d_1, d_2, \ldots, d_n)$ is said to be potentially $S_{L,M}$-graphic (respectively $\overline{S}_{L,M}$)-graphic if there is a realization G of π containing $S_{L,M}$ (respectively $\overline{S}_{L,M}$) as a subgraph. If π has a realization G containing $S_{L,M}$ on those vertices having degree $d_1, d_2, \ldots, d_{L+M}$, then π is potentially $A_{L,M}$-graphic. A non-increasing sequence of non-negative integers $\pi = (d_1, d_2, \ldots, d_n)$ is potentially $A_{L,M}$-graphic if and only if it is potentially $S_{L,M}$-graphic. Let $d_1^{'} \geq d_2^{'} \geq \cdots \geq d_{n-1}^{'}$ be the rearrangement in non-increasing order of $d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n$, then $\pi' = (d_1^{'}, d_2^{'}, \ldots, d_{n-1}^{'})$ is called the residual sequence of π.

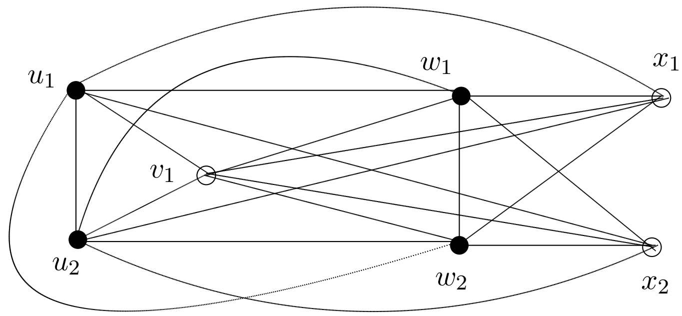

Figure 3 $(S_{3,4} = \overline{S}_{2,1} \vee \overline{S}_{2,2})$

Let $\pi_1, \pi_2, \ldots, \pi_p$ be graphical sequences realizing the graphs $G_1, G_2, \ldots, G_p$. The following result can be seen in [7]. Some more results on split graphs can be found in [8].

Theorem 2.5. *Let π_i of G_i be potentially $\overline{S}_{r_i,s_i}$-graphic, for $i = 1, 2, \ldots, p$. Then*
(1) the graphic sequence π of $G = G_1 \vee G_2 \vee \cdots \vee G_m$ is potentially $S_{L,M}$-graphic, where $L = \sum_{i=1}^{p} r_i$ and $M = \sum_{i=1}^{p} s_i$.
(2) the graphic sequence of $S_{L,M}$ is

$$\pi' = \Big((\sum_{i=1}^{p} (r_i + s_i - 1))^{r_j}, (\sum_{i=1}^{p} r_i + \sum_{i=1, i\neq j}^{p} s_i)^{s_j} \Big),$$

for $j = 1, 2, \cdots, p$
(3)

$$\sigma(\pi') = \Big(\sum_{i=1}^{p} r_i \Big)^2 + 2 \sum_{i=1}^{p} r_i \sum_{j=1}^{p} s_j + s_j \Big(\sum_{i=1, i\neq j}^{p} s_i \Big) - \sum_{i=1}^{p} r_i.$$

The following results are due to Yin [11].

Theorem 2.6. *π is potentially $\overline{A}_{r,s}$-graphic if and only if π_r is graphic.*

Theorem 2.7. *Let $n \geq r + s$ and let $\pi = (d_1, d_2, \ldots, d_n)$ be a non-increasing graphic sequence. If $d_{r+s} \geq 2r + s - 2$, then π is potentially $\overline{A}_{r,s}$-graphic.*

3. **Main results**

We start with the following observations.

If π has a realization G containing K_{r+1} on those vertices having degree $d_1, d_2, \ldots, d_{r+1}$, then π is potentially A_{r+1}-graphic. Rao [9] showed that a non-increasing sequence $\pi = (d_1, d_2, \ldots, d_n)$ is potentially A_{r+1}-graphic if and only if it is potentially K_{r+1}-graphic, Rao [9] considered the problem of characterizing potentially K_{r+1}-graphic sequences and developed a Havel-Hakimi type procedure to determine the maximum clique number of a graph with a given degree sequence π. This procedure can also be used to construct a graph with the degree sequence π and containing K_{r+1} on the first $r+1$ vertices.

Let $n \geq L+1$ and let $\pi = (d_1, d_2, \ldots, d_n)$ be a non-increasing sequence of non negative integers with $d_{L+1} \geq L$. We construct a sequence of sequences $\pi_1, \pi_2, \ldots, \pi_{r_1}, \pi_{r_1+1}, \ldots, \pi_L$ as follows. We first construct the sequence $\pi_1 = (d_2 - 1, d_3 - 1, \ldots, d_{L+1} - 1, d^{(1)}_{L+2}, \ldots, d^{(1)}_n)$ from π by deleting d_1, and reducing the first d_1 terms of π by one and then reordering the last $n - L - 1$ terms to be in non-increasing order. For $2 \leq i \leq L$, we construct $\pi_i = (d_{i+1} - i, d_{i+2} - i, \ldots, d_{L+1} - i, d^{(i)}_{L+2}, \ldots, d^{(i)}_n)$ from $\pi_{i-1} = (d_i - i + 1, d_{i+1} - i + 1, \ldots, d_{L+1} - i + 1, d^{(i-1)}_{L+2}, \ldots, d^{(i-1)}_n)$ by deleting $d_i - i + 1$, reducing the first $d_i - i + 1$ remaining terms of $\pi_i - 1$ by one and then reordering the last $n - L - 1$ terms to be non-increasing.

We have the following observation.

Lemma 3.1. *If π is potentially $A_{L,M}$-graphic then π is potentially $\overline{A}_{L,M}$-graphic.*

Proof. Suppose π is potentially $A_{L,M}$ - graphic. Then there exists a realization G of π containing $S_{L,M}$ as a subgraph and therefore also contains $\overline{S}_{L,M}$, since $\overline{S}_{L,M}$ is a subgraph of $S_{L,M}$. Thus π is potentially $\overline{A}_{L,M}$-graphic. □

Remark 3.2. *The converse of Lemma 3.1 is not true in general. This can be seen as follows.*

Let π be potentially $\overline{A}_{L,M}$-graphic. Take $r_1 = 2, r_2 = 2$ and $s_1 = 1, s_2 = 2$, so that $L = \sum_{i=1}^{2} r_i = 2 + 2 = 5$ and $M = \sum_{i=1}^{2} s_i = 1 + 2 = 3$. Clearly the realization of π contains $\overline{S}_{L,M}$ as a subgraph but it does not contain $S_{L,M}$ as a subgraph, as shown in Figure 1. Therefore π is not potentially $A_{L,M}$-graphic.

The main results of this paper are 3.3 and 3.4.

Theorem 3.3. *π is potentially $A_{L,M}$-graphic if and only if π_L is graphic.*

Proof. Assume that π is potentially $A_{L,M}$- graphic. Then π has a realization G with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ such that $d_G(v_i) = d_i$ for $(1 \leq i \leq n)$ and G contains $S_{L,M}$ on the vertices $v_1, v_2, \ldots, v_{L+M}$, where $L+M \leq n$, so that $V(K_L) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L,+M}\}$. We show that π has a realization G such that v_1 is adjacent to the vertices $v_{L+M+1}, \ldots, v_{d_1+1}$.

If not, we may choose such a realization H of π such that the number of vertices adjacent to v_1 in $\{v_{L+M+1}, \ldots, v_{d_1+1}\}$ is maximum.

Let $v_i \in \{v_{L+M+1}, \ldots, v_{d_1+1}\}$ and $v_1v_i \notin E(H)$ and $v_1v_j \in E(H)$ and let $v_j \in \{v_{d_1+2}, \ldots, v_n\}$. We assume that $d_i \geq d_j$, since the order of i and j can be interchanged, if $d_i = d_j$. Thus there is a vertex v_t, $(t \neq i, j)$ such that $v_iv_t \in E(H)$ and $v_jv_t \notin E(H)$. Construct a graph G from H by deleting the edges v_1v_j and v_iv_t and adding the edges v_1v_i and v_jv_t. This operation does not change the degrees. Therefore G is a realization of π such that $d_G(v_i) = d_i$, for $1 \leq i \leq n$ and G contains $S_{L,M}$ on the vertices $v_1, v_2, \ldots, v_{L+M}$ with $V(K_L) = \{v_1, v_2, \ldots, v_{r_1}, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}$. Also in G, the number of vertices adjacent to v_1 in $\{V_{L+M+1}, \ldots, v_{d_1+1}\}$, is larger than that of H. This contradicts the choice of H. Clearly, π_1 the graphic sequence of $G - v_1$ is potentially $A_{L-1,M}$-graphic. Repeating this procedure, we can see that π_i is potentially $A_{L-i,M}$-graphic successively for $i = 2, 3, \ldots, r_1, r_1 + 1, \ldots, L$. In particular, π_L is graphic.

Conversely, suppose that π_L is graphic and is realized by a graph G_L with the vertex set $V(G_L) = \{v_{L+1}, \ldots, v_n\}$ such that $d_{G_L}(v_i) = d_i$ for $L + 1 \leq i \leq n$. For $i = L, L-1, \ldots, 2, 1$, form G_{i-1} from G_i by adding a new vertex v_i, that is adjacent to each of $v_{i+1}, \ldots, v_{L+M}$ and also to the vertices of G_i with degrees $d^{(i-1)}_{L+M+1} - 1, \ldots, d^{(i-1)}_{d_i+1} - 1$. Then for each i, G_i has degrees given by π_i and G_i contains $S_{L-i,M}$ on $L + M - i$ vertices $v_{i+1}, \ldots, v_{L+M}$ whose degrees are $d_{i+1} - i, \ldots, d_{L+M} - i$ so that $V(K_{L-i}) = \{v_{i+1}, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L+M}\}$ whose degrees are $d_1, d_2, \ldots, d_{L+M}$. So $V(K_L) = \{v_1, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L+M}\}$. In particular, G has degrees given by π and contains $S_{L,M}$ on $L + M$ vertices $v_1, v_2, \ldots, v_{L+M}$ whose degrees are $d_1, d_2, \ldots, d_{L+M}$ so that $V(K_L) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}$. □

Theorem 3.4. *Let $n \geq L+M$ and let $\pi = (d_1, d_2, d_3, \ldots, d_{r_1}, d_{r_1+1}, \ldots, d_n)$ be a non-increasing graphic sequence. If $d_{L+M} \geq 2L + M - 2$, then π is potentially $A_{L,M}$ - graphic.*

Proof. Let $n \geq L + M$ and let $\pi = (d_1, d_2, d_3, \ldots, d_{r_1}, d_{r_1+1}, d_{r_1+2}, \ldots, d_n)$ be a non-increasing graphic sequence. Using the same argument as in Theorem 2.7, π is potentially A_L - graphic. Therefore, we may assume that G is a realization of π with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ such that $d_G(v_i) = d_i$, $(1 \leq i \leq n)$ and G contains K_L on $v_1, v_2, \ldots, v_L$ and

$$t = E_G(\{v_1, v_2, \ldots, v_{r_1}, \ldots, v_L\}, \{v_{L+1}, v_{L+2}, \ldots, v_{L+s_1}, \ldots, v_{L+M}\})$$

(that is, the number of edges between $\{v_1, v_2, \ldots, v_{r_1}, \ldots, v_L\}$ and $\{v_{L+1}, v_{L+2}, \ldots, v_{L+s_1}, \ldots, v_{L+M}\}$) is maximum. If $t = LM + s_1 s_2 + s_j \sum_{i=1}^{j-1} s_i$, for $j = 3, 4, \ldots, p$, then G contains $S_{L,M}$ on $v_1, v_2, \ldots, v_{L+M}$ with $V(K_M) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+s_1}, \ldots, v_{L+M}\}$. In other-words, π is potentially $A_{L,M}$ - graphic. Assume that $t < \{LM + s_1 s_2 + s_j \sum_{i=1}^{j-1} s_i\}$, for $j = 3, 4, \ldots, p$. Then there exists a $v_k \in \{v_1, v_2, \ldots, v_{s_i}\}$ and $v_m \in \{v_{s_i+1}, v_{s_i+2}, \ldots, v_{s_i+s_j}\}, (i \neq j)$ such that $v_k v_m \notin E(G)$. Let

$$A = N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, \ldots, v_{s_i+s_j}\}}(v_k) \setminus N_{G \setminus \{v_1, v_2, \ldots, v_{s_i}\}}(v_m)$$

$$B = N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, \ldots, v_{s_i+s_j}\}}(v_k) \cap N_{G \setminus \{v_1, v_2, \ldots, v_{s_i}\}}(v_m).$$

Then $xy \in E(G)$ for $x \in N_{G \setminus \{v_1, v_2, \ldots, v_{r_i}\}}(v_m)$ and $y \in N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, \ldots, v_{s_i+s_j}\}}(v_k)$. Otherwise, if $xy \notin E(G)$, then $G' = (G \setminus \{v_k y, v_m x\}) \cup \{v_k v_m, xy\}$ is a realization of π and contains $S_{L,M}$ on $v_1, v_2, \ldots, v_{r_1}, v_{r_1+1}, \ldots, v_{L+s_1}, \ldots, v_{L+M}$ with $V(K_L) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}$ such that $E_{G'}(\{v_1, v_2, \ldots, v_L\}, \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}) > t$ which contradicts the choice of G. Thus B is complete. We consider the following two cases.

Case 1. $A = \phi$. Then $2L + M - 2 \leq d_k = d_G(v_k) \leq L + M - 2 + |B|$, and so $|B| \geq L$. Since each vertex in $N_{G \setminus \{v_1, v_2, \ldots, v_L\}}(v_m)$ is adjacent to each vertex in B and $|N_{G \setminus \{v_1, v_2, \ldots, v_L\}}(v_m)| \geq 2L + M - 2 - (L-1) =$

$L+M-1$, it can be easily seen that the induced subgraph of $N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)\cup\{v_m\}$ in G contains $S_{L,M}$ as a subgraph. Thus π is potentially $A_{L,M}$- graphic.

Case 2. $A\neq\phi$. Let $a\in A$. If there are $x,y\in N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)$ such that $xy\notin E(G)$, then $G'=(G\backslash\{v_mx,v_my,v_ka\})\cup\{v_kv_m,av_mxy\}$ is a realization of π and contains $S_{L,M}$ on $v_1,v_2,\ldots,v_{r_1},v_{r_1+1},\ldots,v_{r_1+s_1},\ldots,v_{L+M}$ with $V(K_L)=\{v_1,v_2,\ldots,v_L\}$ and $V(\overline{K}_M)=\{v_{L+1},v_{L+2},\ldots,v_{L+M}\}$ such that $E_{G'}(\{v_1,v_2,\ldots,v_L\},\{v_{L+1},v_{L+2},\ldots,v_{L+M}\})>T$. This contradicts the choice of G and thus it follows that $N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)$ is complete. Since $|N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)|\geq L+M-1$ and $v_mz\in E(G)$ for any $z\in N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)$, it is easy to see that the induced subgraph of $N_{G\backslash\{v_1,v_2,\ldots,v_L\}}(v_m)\cup\{v_m\}$ in G is complete, and so contains $S_{L,M}$ as a subgraph. Thus π is potentially $A_{L,M}$- graphic. □

Remark 3.5. *If $L=r$, $M=s$ and $p=1$, then Theorem 3.3 reduces to Theorem 2.6 and Theorem 3.4 reduces to Theorem 2.7.*

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