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THE CONDITION FOR A SEQUENCE TO BE POTENTIALLY $A_{L,M}$ - GRAPHIC

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ABSTRACT. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n . A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π . The set of all graphic sequences in NS_n is denoted by GS_n . The complete product split graph on L + M vertices is denoted by $\overline{S}_{L,M} = K_L \vee \overline{K}_M$, where K_L and K_M are complete graphs respectively on $L = \sum_{i=1}^p r_i$ and $M = \sum_{i=1}^p s_i$ vertices with r_i and s_i being integers. Another split graph is denoted by $S_{L,M} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} \vee \cdots \vee \overline{S}_{r_p,s_p} =$ $(K_{r_1} \vee \overline{K}_{s_1}) \vee (K_{r_2} \vee \overline{K}_{s_2}) \vee \cdots \vee (K_{r_p} \vee \overline{K}_{s_p})$. A sequence $\pi = (d_1, d_2, \ldots, d_n)$ is said to be potentially $S_{L,M}$ -graphic (respectively $\overline{S}_{L,M}$)-graphic if there is a realization G of π containing $S_{L,M}$ (respectively $\overline{S}_{L,M}$) as a subgraph. If π has a realization G containing $S_{L,M}$ on those vertices having degrees d_1, d_2, \ldots, d_n) is potentially $A_{L,M}$ -graphic. A non-increasing sequence of non-negative integers $\pi = (d_1, d_2, \ldots, d_n)$ is potentially $A_{L,M}$ -graphic if and only if it is potentially $S_{L,M}$ -graphic. In this paper, we obtain the sufficient condition for a graphic sequence to be potentially $A_{L,M}$ -graphic and this result is a generalization of that given by J. H. Yin on split graphs.

1. Introduction

Let G(V, E) be a simple graph (a graph without multiple edges and loops) with n vertices and medges having vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n . A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π . The set of all graphic sequences in NS_n is denoted by GS_n . There are several famous results, Havel and Hakimi [3, 4] and Erdös and Gallai [1] which give necessary and sufficient conditions for a sequence

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 $\pi = (d_1, d_2, \dots, d_n)$ to be the degree sequence of a simple graph G. A combinatorial characterization of degree sequences can be seen in [6]. The disjoint union of the graphs G_1 and G_2 is written as $G_1 \cup G_2$. If $G_1 = G_2 = G$, we write $G_1 \cup G_2$ as 2G. Further K_k , C_k , T_k and P_k respectively denote a complete graph on k vertices, a cycle on k vertices, a tree on k + 1 vertices and a path on k + 1 vertices. A clique is a maximal complete subgraph. If $\pi = (d_1, d_2, \dots, d_n)$ is a graphic sequence, we denote by $\sigma(\pi) = d_1 + d_2 + \cdots + d_n$. A graphic sequence π is potentially H-graphic if there is a realization of π containing H as a subgraph, while π is forcibly H graphic if every realization of π contains H as a subgraph. If π has a realization in which the r vertices of largest degree induce a clique, then π is said to be potentially A_r -graphic. It is shown in [2] that if π is a graphic sequence with a realization G containing H as a subgraph, then there is a realization G of π containing H with the vertices of H having |V(H)| largest degree of π . We know that a graphic sequence π is potentially K_{k+1} -graphic if and only if π is potentially A_{k+1} -graphic [10].

In this paper, we obtain the sufficient condition for a graphic sequence to be potentially $A_{L,M}$ -graphic and this result is a generalization of that given by Yin [11] on split graphs. The main results of this paper are Theorems 3.3 and 3.4.

2. Definitions and preliminaries

We have the following definitions.

Definition 2.1. The join (complete product) of two graphs G_1 and G_2 is a graph $G = G_1 \vee G_2$ with vertex set $V(G_1) \cup V(G_2)$ and the edge set consisting of all the edges of G_1 and G_2 together with the edges joining each vertex of G_1 with every vertex of G_2 .

Definition 2.2. [9]. The split graph of the complete graphs K_r and K_s , denoted by $\overline{S}_{r,s}$, is the graph $K_r \vee \overline{K}_s$ having r + s vertices, where \overline{K}_s is the complement of K_s . We note that $\overline{S}_{r,1} = K_{r+1}$. For example, let r = 3 and s = 4, we form the split graph $\overline{S}_{3,4} = K_3 \vee \overline{K}_4$. We take $V(K_3) = \{v_1, v_2, v_3\}$ and $V(\overline{K}_4) = \{u_1, u_2, u_3, u_4\}$. The resulting split graph $\overline{S}_{3,4}$ on seven vertices is shown in Figure 1.



Let r_1, r_2, \ldots, r_p and s_1, s_2, \ldots, s_p be positive integers and let $\sum_{i=1}^p r_i = L$ and $\sum_{i=1}^p s_i = M$. The split graph (by Definition 2.1) in this case is denoted by $\overline{S}_{L,M} = K_L \vee \overline{K}_M$.

Definition 2.3. Let $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$ be split graphs, respectively with $r_1 + s_1, r_2 + s_2, \ldots, r_p + s_p$ vertices. The split graph in this case, denoted by $S_{L,M}$, is the complete product of $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$. We write

$$S_{L,M} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} \vee \dots \vee \overline{S}_{r_p,s_p}$$
$$= (K_{r_1} \vee \overline{K}_{s_1}) \vee (K_{r_2} \vee \overline{K}_{s_2}) \vee \dots \vee (K_{r_p} \vee \overline{K}_{s_p}).$$

Clearly $S_{L,M}$ has vertex set $\bigcup_{i=1}^{p} V(\overline{S}_{r_i,s_i})$ and the edge set consists of all the edges of $\overline{S}_{r_1,s_1}, \overline{S}_{r_2,s_2}, \ldots, \overline{S}_{r_p,s_p}$ together with the edges joining each vertex of \overline{S}_{r_i,s_i} with every vertex of \overline{S}_{r_j,s_j} for every i, j with $i \neq j$.

Remark 2.4. $\overline{S}_{L,M}$ is always a subgraph of $S_{L,M}$.

For example, let $r_1 = 2$, $r_2 = 2$, $s_1 = 1$ and $s_2 = 2$. Take $L = r_1 + r_2 = 2 + 2 = 4$, and $M = s_1 + s_2 = 1 + 2 = 3$. We construct the split graphs $\overline{S}_{L,M}$ and $S_{L,M}$ as follows.

We have, $\overline{S}_{L,M} = \overline{S}_{3,4} = K_3 \vee \overline{K}_4$. This is shown in Figure 1.

We have $\overline{S}_{r_1,s_1} = \overline{S}_{2,1} = K_2 \vee \overline{K_1}$ and $\overline{S}_{r_2,s_2} = \overline{S}_{2,2} = K_2 \vee \overline{K_2}$. We form the complete product split graph $S_{L,M} = S_{3,4} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} = \left(K_2 \vee \overline{K_1}\right) \vee \left(K_2 \vee \overline{K_2}\right)$. We take the vertex set of $\overline{S}_{r_1,s_1} = \overline{S}_{2,1}$ as $\{u_1, u_2, v_1\}$, and the vertex set of $\overline{S}_{r_2,s_2} = \overline{S}_{2,2}$ as $\{w_1, w_2, x_1, x_2\}$. The resulting complete product split graph $S_{L,M} = S_{3,4}$ on seven vertices is shown in Figure 3.



A sequence $\pi = (d_1, d_2, \ldots, d_n)$ is said to be potentially $S_{L,M}$ -graphic (respectively $\overline{S}_{L,M}$)-graphic if there is a realization G of π containing $S_{L,M}$ (respectively $\overline{S}_{L,M}$) as a subgraph. If π has a realization Gcontaining $S_{L,M}$ on those vertices having degree $d_1, d_2, \ldots, d_{L+M}$, then π is potentially $A_{L,M}$ -graphic. A non-increasing sequence of non-negative integers $\pi = (d_1, d_2, \ldots, d_n)$ is potentially $A_{L,M}$ -graphic if and only if it is potentially $S_{L,M}$ -graphic. Let $d'_1 \geq d'_2 \geq \cdots \geq d'_{n-1}$ be the rearrangement in nonincreasing order of $d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n$, then $\pi' = (d'_1, d'_2, \ldots, d'_{n-1})$ is called the residual sequence of π .



Figure 3 $(S_{3,4} = \overline{S}_{2,1} \lor \overline{S}_{2,2})$

Let $\pi_1, \pi_2, \ldots, \pi_p$ be graphical sequences realizing the graphs G_1, G_2, \ldots, G_p . The following result can be seen in [7]. Some more results on split graphs can be found in [8].

Theorem 2.5. Let π_i of G_i be potentially \overline{S}_{r_i,s_i} -graphic, for i = 1, 2, ..., p. Then (1) the graphic sequence π of $G = G_1 \vee G_2 \vee \cdots \vee G_m$ is potentially $S_{L,M}$ -graphic, where $L = \sum_{i=1}^p r_i$ and $M = \sum_{i=1}^p s_i$. (2) the graphic sequence of $S_{L,M}$ is $\pi' = \left((\sum_{i=1}^p (r_i + s_i - 1))^{r_j}, (\sum_{i=1}^p r_i + \sum_{i=1, i \neq j}^p s_i)^{s_j} \right),$

for $j = 1, 2, \cdots, p$ (3)

$$\sigma(\pi') = \left(\sum_{i=1}^{p} r_i\right)^2 + 2\sum_{i=1}^{p} r_i \sum_{j=1}^{p} s_j + s_j \left(\sum_{i=1, i \neq j}^{p} s_i\right) - \sum_{i=1}^{p} r_i.$$

The following results are due to Yin [11].

Theorem 2.6. π is potentially $\overline{A}_{r,s}$ -graphic if and only if π_r is graphic.

Theorem 2.7. Let $n \ge r + s$ and let $\pi = (d_1, d_2, \ldots, d_n)$ be a non-increasing graphic sequence. If $d_{r+s} \ge 2r + s - 2$, then π is potentially $\overline{A}_{r,s}$ -graphic.

3. Main results

We start with the following observations.

If π has a realization G containing K_{r+1} on those vertices having degree $d_1, d_2, \ldots, d_{r+1}$, then π is potentially A_{r+1} -graphic. Rao [9] showed that a non-increasing sequence $\pi = (d_1, d_2, \ldots, d_n)$ is potentially A_{r+1} -graphic if and only if it is potentially K_{r+1} -graphic, Rao [9] considered the problem

of characterizing potentially K_{r+1} -graphic sequences and developed a Havel-Hakimi type procedure to determine the maximum clique number of a graph with a given degree sequence π . This procedure can also be used to construct a graph with the degree sequence π and containing K_{r+1} on the first r + 1vertices.

Let $n \ge L+1$ and let $\pi = (d_1, d_2, \ldots, d_n)$ be a non-increasing sequence of non negative integers with $d_{L+1} \ge L$. We construct a sequence of sequences $\pi_1, \pi_2, \ldots, \pi_{r_1}, \pi_{r_1+1}, \ldots, \pi_L$ as follows. We first construct the sequence $\pi_1 = (d_2 - 1, d_3 - 1, \ldots, d_{L+1} - 1, d_{L+2}^{(1)}, \ldots, d_n^{(1)})$ from π by deleting d_1 , and reducing the first d_1 terms of π by one and then reordering the last n - L - 1 terms to be in non-increasing order. For $2 \le i \le L$, we construct $\pi_i = (d_{i+1} - i, d_{i+2} - i, \ldots, d_{L+1} - i, d_{L+2}^{(i)}, \ldots, d_n^{(i)})$ from $\pi_{i-1} = (d_i - i + 1, d_{i+1} - i + 1, \ldots, d_{L+1} - i + 1, d_{L+2}^{(i-1)}, \ldots, d_n^{(i-1)})$ by deleting $d_i - i + 1$, reducing the first $d_i - i + 1$ remaining terms of $\pi_i - 1$ by one and then reordering the last n - L - 1 terms to be non-increasing.

We have the following observation.

Lemma 3.1. If π is potentially $A_{L,M}$ -graphic then π is potentially $\overline{A}_{L,M}$ -graphic.

Proof. Suppose π is potentially $A_{L,M}$ - graphic. Then there exists a realization G of π containing $S_{L,M}$ as a subgraph and therefore also contains $\overline{S}_{L,M}$, since $\overline{S}_{L,M}$ is a subgraph of $S_{L,M}$. Thus π is potentially $\overline{A}_{L,M}$ -graphic.

Remark 3.2. The converse of Lemma 3.1 is not true in general. This can be seen as follows.

Let π be potentially $\overline{A}_{L,M}$ -graphic. Take $r_1 = 2, r_2 = 2$ and $s_1 = 1, s_2 = 2$, so that $L = \sum_{i=1}^{2} r_i = 2 + 2 = 5$ and $M = \sum_{i=1}^{2} s_i = 1 + 2 = 3$. Clearly the realization of π contains $\overline{S}_{L,M}$ as a subgraph but it does not contain $S_{L,M}$ as a subgraph, as shown in Figure 1. Therefore π is not potentially $A_{L,M}$ -graphic.

The main results of this paper are 3.3 and 3.4.

Theorem 3.3. π is potentially $A_{L,M}$ -graphic if and only if π_L is graphic.

Proof. Assume that π is potentially $A_{L,M}$ -graphic. Then π has a realization G with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ such that $d_G(v_i) = d_i$ for $(1 \le i \le n)$ and G contains $S_{L,M}$ on the vertices $v_1, v_2, \ldots, v_{L+M}$, where $L+M \le n$, so that $V(K_L) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L,+M}\}$. We show that π has a realization G such that v_1 is adjacent to the vertices $v_{L+M+1}, \ldots, v_{d_1+1}$.

If not, we may choose such a realization H of π such that the number of vertices adjacent to v_1 in $\{v_{L+M+1}, \ldots, v_{d_1+1}\}$ is maximum.

Let $v_i \in \{v_{L+M+1}, \ldots, v_{d_1+1}\}$ and $v_1v_i \notin E(H)$ and $v_1v_j \in E(H)$ and let $v_j \in \{v_{d_1+2}, \ldots, v_n\}$. We assume that $d_i \geq d_j$, since the order of i and j can be interchanged, if $d_i = d_j$. Thus there is a vertex v_t , $(t \neq i, j)$ such that $v_iv_t \in E(H)$ and $v_jv_t \notin E(H)$. Construct a graph G from H by deleting the edges v_1v_j and v_iv_t and adding the edges v_1v_i and v_jv_t . This operation does not change the degrees. Therefore G is a realization of π such that $d_G(v_i) = d_i$, for $1 \leq i \leq n$ and G contains $S_{L,M}$ on the vertices $v_1, v_2, \ldots, v_{L+M}$ with $V(K_L) = \{v_1, v_2, \ldots, v_{r_1}, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}$. Also in G, the number of vertices adjacent to v_1 in $\{V_{L+M+1}, \ldots, v_{d_1+1}\}$, is larger than that of H. This contradicts the choice of H. Clearly, π_1 the graphic sequence of $G - v_1$ is potentially $A_{L-1,M}$ graphic. Repeating this procedure, we can see that π_i is potentially $A_{L-i,M}$ -graphic successively for $i = 2, 3, \ldots, r_1, r_1 + 1, \ldots, L$. In particular, π_L is graphic.

Conversely, suppose that π_L is graphic and is realized by a graph G_L with the vertex set $V(G_L) = \{v_{L+1}, \ldots, v_n\}$ such that $d_{G_L}(v_i) = d_i$ for $L+1 \leq i \leq n$. For $i = L, L-1, \ldots, 2, 1$, form G_{i-1} from G_i by adding a new vertex v_i , that is adjacent to each of v_{i+1}, \ldots, v_{L+M} and also to the vertices of G_i with degrees $d_{L+M+1}^{(i-1)} - 1, \ldots, d_{d_i+1}^{(i-1)} - 1$. Then for each i, G_i has degrees given by π_i and G_i contains $S_{L-i,M}$ on L + M - i vertices v_{i+1}, \ldots, v_{L+M} whose degrees are $d_{i+1} - i, \ldots, d_{L+M} - i$ so that $V(K_{L-i}) = \{v_{i+1}, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L+M}\}$ whose degrees are $d_1, d_2, \ldots, d_{L+M}$. So $V(K_L) = \{v_1, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, \ldots, v_{L+M}\}$. In particular, G has degrees given by π and contains $S_{L,M}$ on L + M vertices $v_1, v_2, \ldots, v_{L+M}$ whose degrees are $d_1, d_2, \ldots, d_{L+M}$ so that $V(K_L) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}$.

Theorem 3.4. Let $n \ge L+M$ and let $\pi = (d_1, d_2, d_3, \ldots, d_{r_1}, d_{r_1+1}, \ldots, d_n)$ be a non-increasing graphic sequence. If $d_{L+M} \ge 2L + M - 2$, then π is potentially $A_{L,M}$ - graphic.

Proof. Let $n \ge L + M$ and let $\pi = (d_1, d_2, d_3, \dots, d_{r_1}, d_{r_1+1}, d_{r_1+2}, \dots, d_n)$ be a non-increasing graphic sequence. Using the same argument as in Theorem 2.7, π is potentially A_L - graphic. Therefore, we may assume that G is a realization of π with the vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ such that $d_G(v_i) = d_i$, $(1 \le i \le n)$ and G contains K_L on v_1, v_2, \dots, v_L and

$$t = E_G(\{v_1, v_2, \dots, v_{r_1}, \dots, v_L\}, \{v_{L+1}, v_{L+2}, \dots, v_{L+s_1}, \dots, v_{L+M}\})$$

(that is, the number of edges between $\{v_1, v_2, \ldots, v_{r_1}, \ldots, v_L\}$ and $\{v_{L+1}, v_{L+2}, \ldots, v_{L+s_1}, \ldots, v_{L+M}\}$) is maximum. If $t = LM + s_1s_2 + s_j \sum_{i=1}^{j-1} s_i$, for $j = 3, 4, \ldots, p$, then G contains $S_{L,M}$ on $v_1, v_2, \ldots, v_{L+M}$ with $V(K_M) = \{v_1, v_2, \ldots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \ldots, v_{L+s_1}, \ldots, v_{L+M}\}$. In other-words, π is potentially $A_{L,M}$ - graphic. Assume that $t < \{LM + s_1s_2 + s_j \sum_{i=1}^{j-1} s_i\}$, for $j = 3, 4, \ldots, p$. Then there exists a $v_k \in \{v_1, v_2, \ldots, v_{s_i}\}$ and $v_m \in \{v_{s_i+1}, v_{s_i+2}, \ldots, v_{s_i+s_j}\}, (i \neq j)$ such that $v_k v_m \notin E(G)$. Let

$$A = N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, \dots, v_{s_i+s_j}\}}(v_k) \setminus N_{G \setminus \{v_1, v_2, \dots, v_{s_i}\}}(v_m)$$

$$B = N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, \dots, v_{s_i+s_j}\}}(v_k) \cap N_{G \setminus \{v_1, v_2, \dots, v_{s_i}\}}(v_m)$$

Then $xy \in E(G)$ for $x \in N_{G \setminus \{v_1, v_2, ..., v_{r_i}\}}(v_m)$ and $y \in N_{G \setminus \{v_{s_i+1}, v_{s_i+2}, ..., v_{s_i+s_j}\}}(v_k)$. Otherwise, if $xy \notin E(G)$, then $G' = (G \setminus \{v_k y, v_m x\}) \cup \{v_k v_m, xy\}$ is a realization of π and contains $S_{L,M}$ on $v_1, v_2, ..., v_{r_1}, v_{r_1+1}, ..., v_{L+s_1}, ..., v_{L+M}$ with $V(K_L) = \{v_1, v_2, ..., v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, ..., v_{L+M}\}$

 \ldots, v_{L+M} such that $E_{G'}(\{v_1, v_2, \ldots, v_L\}, \{v_{L+1}, v_{L+2}, \ldots, v_{L+M}\}) > t$ which contradicts the choice of G. Thus B is complete. We consider the following two cases.

Case 1. $A = \phi$. Then $2L + M - 2 \le d_k = d_G(v_k) \le L + M - 2 + |B|$, and so $|B| \ge L$. Since each vertex in $N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)$ is adjacent to each vertex in B and $|N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)| \ge 2L + M - 2 - (L - 1) = 0$

L + M - 1, it can be easily seen that the induced subgraph of $N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m) \cup \{v_m\}$ in G contains $S_{L,M}$ as a subgraph. Thus π is potentially $A_{L,M}$ - graphic.

Case 2. $A \neq \phi$. Let $a \in A$. If there are $x, y \in N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)$ such that $xy \notin E(G)$, then $G' = (G \setminus \{v_m x, v_m y, v_k a\}) \cup \{v_k v_m, av_m xy\}$ is a realization of π and contains $S_{L,M}$ on $v_1, v_2, \dots, v_{r_1}, v_{r_1+1}, \dots, v_{r_1+s_1}, \dots, v_{L+M}$ with $V(K_L) = \{v_1, v_2, \dots, v_L\}$ and $V(\overline{K}_M) = \{v_{L+1}, v_{L+2}, \dots, v_{L+M}\}$ such that $E_{G'}(\{v_1, v_2, \dots, v_L\}, \{v_{L+1}, v_{L+2}, \dots, v_{L+M}\}) > T$. This contradicts the choice of G and thus it follows that $N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)$ is complete. Since $|N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)| \ge L + M - 1$ and $v_m z \in E(G)$ for any $z \in N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m)$, it is easy to see that the induced subgraph of $N_{G \setminus \{v_1, v_2, \dots, v_L\}}(v_m) \cup \{v_m\}$ in G is complete, and so contains $S_{L,M}$ as a subgraph. Thus π is potentially $A_{L,M}$ - graphic. \Box

Remark 3.5. If L = r, M = s and p = 1, then Theorem 3.3 reduces to Theorem 2.6 and Theorem 3.4 reduces to Theorem 2.7.

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