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A FAMILY OF t -REGULAR SELF-COMPLEMENTARY k -HYPERGRAPHS

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ABSTRACT. We use the recursive method of construction large sets of t -designs given by Qiu-rong Wu (A note on extending t -designs, *Australas. J. Combin.*, **4** (1991) 229–235.), and present a similar method for constructing t -subset-regular self-complementary k -uniform hypergraphs of order v . As an application we show the existence of a new family of 2-subset-regular self-complementary 4-uniform hypergraphs with $v = 16m + 3$.

1. Introduction

Let t, k and v be positive integers such that $t \leq k \leq v$. Let X be a set of size v (or a v -set, called point set) and let $P_i(X)$, $0 < i \leq t$, be the set of all i -subsets of X . A t - (v, k, λ) design (briefly a t -design) is a pair $\mathbf{D}=(V, D)$ in which D is a collection of elements of $P_k(V)$ (called blocks) such that every t -subset of V appears in exactly λ blocks. Two t - (v, k, λ) designs (V_1, D_1) and (V_2, D_2) are said to be *isomorphic* if there is a bijection $\sigma : V_1 \rightarrow V_2$ such that $\sigma(D_1) = D_2$. Any isomorphism of \mathbf{D} into itself is called an *automorphism*. The set of all automorphisms of D forms a subgroup of $Sym(X)$ and is called the automorphism group of the design denoted by $Aut(D)$. If G is a subgroup of $Aut(D)$, we say that D is G -invariant. Let $N \geq 1$ be a positive integer. A *large set of t - (v, k, λ) design* of size N , denoted by $LS[N](t, k, v)$, is a partition of $P_k(V)$ into N disjoint t - (v, k, λ) designs.

A k -uniform hypergraph of order v is an ordered pair $H = (V, E)$, where $V = V(H)$ is a v -set (called vertex set) and $E = E(H)$ is a subset of $P_k(V)$ (called edge set). We call a k -uniform hypergraph

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simply a k -hypergraph [4]. Two k -hypergraphs H_1 and H_2 are *isomorphic*, if there is a bijection $\theta : V(H_1) \rightarrow V(H_2)$, such that $\{x_1, x_2, \dots, x_k\} \in E(H_1)$, if and only if $\{\theta(x_1), \theta(x_2), \dots, \theta(x_k)\} \in E(H_2)$. A k -hypergraph H is called *self-complementary* if H is isomorphic with its *complement* H^c , where H^c is the hypergraph defined by $H^c = (V, P_k(V) \setminus E(H))$. An *antimorphism* is an isomorphism between a hypergraph and its complement.

A k -hypergraph H of order v is *t -subset-regular* (or *t -regular*) if there exists a positive integer λ (called the t -valence of H), such that each element of $P_t(V)$ is a subset of exactly λ elements of $E(H)$. Henceforth we denote such a structure by $\text{RHG}(t, k, v)$. Moreover, if H is a self-complementary $\text{RHG}(t, k, v)$, then H and H^c form the large set of t -designs $\text{LS}[2](t, k, v)$ with the additional property that these two designs are isomorphic[1]. Henceforth we denote this structure by $\text{SRHG}(t, k, v)$. An easy counting argument shows that a necessary condition for the existence of an $\text{SRHG}(t, k, v)$ is that $\binom{v-i}{k-i}$ is an even integer for all $i = 0, 1, \dots, t$.

In what follows first we mention required notation and results from the literature. First note the following theorem [3] which gives a necessary condition for the existence of $\text{LS}[2](t, k, v)$, in terms of some congruence relations. Let m and n be positive integers, then by (m/n) we denote the remainder of division m by n .

Theorem 1.1. *If there exists an $\text{LS}[2](t, k, v)$, then there exists an integer l such that $t \leq (v/2^l) < (k/2^l)$.*

We may restate the above theorem as a necessary condition for the existence of $\text{SRHG}(t, k, v)$. A more clear version of this condition in the case $t = 2$ and $k = 4$ is mentioned below

Corollary 1.2. *If there exist an $\text{SRHG}(2, 4, v)$, then $v \equiv 2, 3 \pmod{8}$.*

Proof. Let $l = 3$ in Theorem 1.1. □

Remark 1.3. *Let $H = (V(H), E(H))$ be an $\text{SRHG}(t, k, v)$ and $x \in V(H)$ and suppose that θ is its antimorphism with x as a fixed point. We define*

$$H^d(x) = \{e \setminus \{x\} | x \in e \in E(H)\}.$$

then $H^d(x)$ is an $\text{SRHG}(t-1, k-1, v-1)$ and is called the derived hypergraph (the restriction of θ on $\{V(H) \setminus \{x\}\}$ is its antimorphism) [1].

Constructing large sets of t -designs is divided in two categories: direct methods and recursive methods. The most common direct method is based on the concept of group action in group theory. This is a brief review: let G be a subgroup of $\text{Sym}(X)$ and let T_1, T_2, \dots, T_s and K_1, K_2, \dots, K_r (for positive integers r and s) be the orbits of $P_t(X)$ and $P_k(X)$ under the action of G , respectively. Then for a fixed $T \in T_i$, the number of $K \in K_j$ with $T \subseteq K$ is independent of any representative T_i of T . We denote this number by a_{ij} . The Kramer-Mesner matrix is the $s \times r$ matrix $A_{t,k}^v(G)$ whose

(i, j) -th entry is a_{ij} . The following theorem, due to Kramer and Mesner [6] gives a direct method to find G -invariant designs.

Theorem 1.4. *There exists a G -invariant t -(v, k, λ) design if and only if there exists a vector $u \in \{0, 1\}^r$ satisfying the equation*

$$A_{t,k}^v(G)u = \lambda j \tag{1.1}$$

where j is the s -dimensional all one vector.

Remark 1.5. *We may adapt the same method for finding large sets. We pick up one from the set of solutions of u and remove the corresponding columns from $A_{t,k}^v(G)$. The resulting matrix $A_{t,k}^v(G)$ is used in a similar way to find designs via the equation $A_{t,k}^v(G)u' = \lambda j$. We repeat the procedure until all the orbits on k -subsets are used [2].*

Remark 1.6. *Similar to the above remark, one can use Theorem 1.4 to construct SRHG (t, k, v). In this case, we need only one solution for equation (1.1). Note that in order to have self-complementary in the solution, half of elements of each orbits K_j alternately must be chosen. So each orbit K_j should be divided into two sets K'_j and K''_j such that elements of K_j alternately be in them (the same process for each T_i). Hence to construct Kramer-Mesner matrix, we should use K'_j and K''_j instead of K_j . In each solution for equation (1.1), we choose exactly one of the columns correspond to K'_j and K''_j .*

When the block set D of a design is G -invariant for some abelian group G , then one needs only to list a part of the blocks, called *starter blocks*, in order to obtain D . In other words starter blocks are a set of orbit representations in D under the action of G .

Now we extend one the of useful recursive construction of large sets of t -designs to present a similar method for constructing t -subset-regular self-complementary k -hypergraphs. Qiu-rong Wu [7] proved the following theorem for extending $LS[N](t, k, v)$.

Theorem 1.7. *Suppose there are $LS[N](t, k, v_1)$, $LS[N](t, k, v_2)$, $LS[N](k - 2, k - 1, v_1 - 1)$ and $LS[N](k - 2, k - 1, v_2 - 1)$. Then there exists $LS[N](t, k, v_1 + v_2 - k + 1)$.*

2. Main Results

In this section we generalize the method given in Theorem 1.7 to present an SRHG(t, k, v) and use it to construct a family of t -regular self-complementary k -hypergraphs.

Notation. Let β_1 and β_2 be the edge sets of any two k -hypergraphs, we consider the following notation defined in [7]:

$$\beta_1 \odot \beta_2 = \{A \cup B; A \in \beta_1, B \in \beta_2\}.$$

Theorem 2.1. *Suppose that*

- H_0 is SRHG(t, k, v_1) with an antimorphism having at least $(k-1)$ fixed points,
- H_1 is SRHG($k - 2, k - 1, v_1 - 1$) with an antimorphism having at least $(k-2)$ fixed points,

- H_k is $SRHG(t, k, v_2)$ with an antimorphism having at least $(k-1)$ fixed points,
- H_{k-1} and H_{k-1}^c are $RHG(k-2, k-1, v_2-1)$ such that they have a common automorphism with at least $(k-2)$ fixed points.

Then there exists an $SRHG(t, k, v_1 + v_2 - k + 1)$.

Proof. Let $X = \{1, 2, \dots, v_1 + v_2 - k + 1\}$, $X_j = \{1, 2, \dots, v_1 - j\}$ for $j = 0, 1, \dots, k - 1$ and $Y_j = \{v_1 + 2 - j, v_1 + 3 - j, \dots, v_1 + v_2 - k + 1\}$ for $j = 1, 2, \dots, k$. Note that $X_j \cup Y_j = X - \{v_1 + 1 - j\}$ for $j = 1, 2, \dots, k - 1$. Let $\theta \in Sym(X)$ and let $\{v_1 - k + 2, v_1 - k + 3, \dots, v_1\}$ be the set of all fixed points of θ . Consider $H_0 = (X_0, B_{1,0})$, $H_0^c = (X_0, B_{2,0})$, $H_1 = (X_1, B_{1,1})$, $H_1^c = (X_1, B_{2,1})$, $H_k = (Y_k, B'_{1,k})$, $H_k^c = (X_0, B'_{2,k})$, $H_{k-1} = (Y_{k-1}, B'_{1,k-1})$ and $H_{k-1}^c = (Y_{k-1}, B'_{2,k-1})$ such that the restriction of θ on X_0 , X_1 and Y_k be antimorphism of H_0 , H_1 and H_k , respectively. Also let the restriction of θ on Y_{k-1} be automorphism of H_{k-1} and H_{k-1}^c . Let j be an integer in $0 < j < k$. Invoking Remark 1.3, we delete the points $\{v_1 + 1 - j, v_1 + 2 - j, \dots, v_1 - 1\}$ from H_1 and H_1^c and obtain the corresponding derived hypergraphs $H_j = (X_j, B_{1,j})$ and $H_j^c = (X_j, B_{2,j})$, which together form an $SRHG(k-1-j, k-j, v_1-j)$.

Similarly, consider $H_{k-1} = (Y_{k-1}, B'_{1,k-1})$ and $H_{k-1}^c = (Y_{k-1}, B'_{2,k-1})$. Then by deleting the points $\{v_1 + 3 - k, v_1 + 4 - k, \dots, v_1 + 1 - j\}$, for each j in $1 < j < k$, we obtain two corresponding derived hypergraphs $H_{k-j} = (Y_{k-j}, B'_{1,k-j})$ and $H_{k-j}^c = (Y_{k-j}, B'_{2,k-j})$. By Remark 1.3, these two hyperpergraphs are $RHG(j-1, j, v_2-k+j)$. Now let

$$D = B_{1,0} \cup B'_{1,k} \cup [(B_{1,1} \odot B'_{1,1}) \cup (B_{2,1} \odot B'_{2,1})] \cup \dots \cup [(B_{1,k-1} \odot B'_{1,k-1}) \cup (B_{2,k-1} \odot B'_{2,k-1})]$$

$$D' = B_{2,0} \cup B'_{2,k} \cup [(B_{1,1} \odot B'_{2,1}) \cup (B_{2,1} \odot B'_{1,1})] \cup \dots \cup [(B_{1,k-1} \odot B'_{2,k-1}) \cup (B_{2,k-1} \odot B'_{1,k-1})].$$

By argument given in the proof of theorem 1.7, two designs (X, D) and (X, D') together form an $LS[2](t, k, v_1 + v_2 - k + 1)$. On the other hand, θ maps $(B_{1,i} \odot B'_{1,i})$ into $(B_{2,i} \odot B'_{1,i})$. Also θ maps $(B_{2,i} \odot B'_{2,i})$ into $(B_{1,i} \odot B'_{2,i})$ for all $0 < i < k$. It is not difficult to show that θ maps D to D' . This implies that (X, D) is an $SRHG(t, k, v_1 + v_2 - k + 1)$ and this completes the proof. \square

3. 2-Regular Self-Complementary 4-Hypergraphs

In this section, we apply Theorem 3.4 to obtain some new results on $SRHG(2,4,v)$. First note that in 1992 W. Kocay[5] proved the following for 3-hypergraphs.

Theorem 3.1. *A permutation θ is an antimorphism for a 3-hypergraph if and only if either:*

- i.: every cycle of θ has even length, or
- ii.: θ has 1 or 2 fixed points and the lengths of its other cycles are multiples of 4.

Antimorphisms of 4-hypergraphs are characterized in [8], as follows:

Theorem 3.2. *A permutation θ is an antimorphism for a 4-hypergraph if and only if one of the following cases is satisfied:*

- i.: the length of every cycle of θ is a multiple of 8,
- ii.: θ has 1, 2 or 3 fixed points, and the lengths of its other cycles are multiples of 8.

- iii.: θ has 1 cycle of length 2, and the lengths of its other cycles are multiples of 8,
- iv.: θ has 1 fixed point, 1 cycle of length 2 and the lengths of its other cycles are multiples of 8,
- v.: θ has 1 cycle of length 3 and the lengths of its other cycles are multiples of 8.

Note that SRHG(2, 3, v) for all admissible values of v are presented in [4] using antimorphisms without any fixed points. Now, we prove the existence of SRHG(2, 3, $16m + 2$) for all $m \geq 2$ with antimorphisms having 2 fixed points. First let $V = \{0, 1, \dots, 19\}$ and $\theta_1, \theta_2 \in Sym(V)$, where $\theta_1 = (0\ 1\ 2\ 3\ 4\ 5\ 6\ 7)(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15)(16)(17)(18)$ and θ_2 is the restriction of θ_1 on $V \setminus \{18\}$.

Lemma 3.3. *If $m \geq 1$, then there exists an SRHG(2, 3, $16m + 2$) with an antimorphism having two fixed points.*

Proof. The proof is by induction on m . Invoking Remark 1.6 we construct and present an SRHG(2, 3, 18) in table 2 and θ_2 is one of its antimorphisms. In this table every 3-subset is a representative of one orbit of $P_3(V)$ under the action of θ_2^2 , therefore for $m = 1$ there is nothing to prove. So let $m = l > 1$. Let, by induction, there exist an SRHG(2, 3, $16(l - 1) + 2$) with an antimorphism (called θ) having 2 fixed points. Applying Remark 1.3 if we delete a fixed point of θ , then we have a SRHG(1, 2, $16(l - 1) + 1$) such that the restriction of θ is one of its antimorphisms. Note that RHG(1, 2, 17) is given in table 1 and that one of its automorphisms (called θ_3) is obtained by restriction of θ_1 on $V - \{17, 18\}$. In this table every 2-subset is a representative of one orbit of $P_2(V)$ under the action of θ_3 . The complement of this hypergraph is also an RHG(1, 2, 17) with θ_3 as an its automorphisms. So Theorem 3.4 implies the existence of an SRHG(2, 3, $16m + 2$) for $m \geq 2$ with an antimorphism having two fixed points. □

We use the above results to prove the existence of a new family of SRHG(2, 4, v).

Corollary 3.4. *If $m \geq 1$, then there exists SRHG(2, 4, $16m + 3$).*

Proof. The proof is by induction on m . For $m = 1$, one SRHG(2, 4, 19) with an automorphism θ_1 is given in table 3. In this table every 4-subset is a representative of one orbit of $P_4(V)$ under the action of θ_1^2 (by Remark 1.6). Similarly in table 4, one RHG(2, 3, 18) is given. In this case θ_2 is an automorphism of this hypergraph and its complement. Note that the complement of this hypergraph is also an RHG(2, 3, 18). In this table every 3-subset is a representative of one orbit of $P_3(V)$ under the action of θ_2 (constructed by applying Remark 1.5). Now let $m = l > 1$ and let, by the induction the existence of an SRHG(2, 4, $16(l - 1) + 3$). Then Lemma 3.3 shows the existence of an SRHG(2, 3, $16(l - 1) + 2$) and the claim follows from Theorem 3.4. □

Table 1. Representative of orbits under action of θ_3 to construct RHG(1, 2, 17)

0, 2	0, 4	0, 8	0, 9	0, 10	0, 11	0, 16	8, 9	8, 10
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Table 2. Representative of orbits under action of θ_2^2 to construct SRHG(2, 3, 18)

0, 1, 2	0, 10, 15	1, 3, 16	8, 9, 11	0, 8, 16	1, 2, 15	0, 12, 15	0, 3, 17	9, 12, 17	1, 11, 13	0, 2, 13	0, 15, 17	0, 8, 13
1, 2, 12	1, 12, 17	0, 3, 14	9, 11, 16	0, 9, 16	1, 3, 11	1, 15, 16	1, 9, 11	0, 1, 8	0, 11, 14	1, 4, 12	8, 9, 17	0, 9, 13
0, 2, 5	0, 13, 16	0, 4, 16	1, 2, 5	0, 10, 17	0, 3, 8	9, 10, 14	0, 9, 10	0, 1, 16	0, 12, 17	1, 5, 10	8, 12, 17	1, 11, 15
1, 3, 8	8, 9, 10	1, 8, 9	0, 1, 13	0, 12, 14	1, 4, 16	8, 11, 16	1, 11, 12	1, 3, 13	1, 8, 16	1, 9, 13	0, 1, 10	1, 12, 16
1, 4, 14	8, 10, 13	1, 8, 10	0, 2, 9	1, 8, 15	1, 9, 10	0, 1, 6	1, 12, 14	1, 4, 11	8, 9, 16	1, 10, 13	1, 3, 5	1, 8, 14
0, 4, 11	1, 2, 4	1, 11, 16	1, 3, 17	8, 9, 12	1, 9, 17	1, 2, 8	0, 12, 16	0, 4, 8	8, 12, 16	0, 10, 13	1, 3, 15	0, 16, 17
1, 9, 15	1, 2, 13	1, 12, 13	0, 3, 15	9, 11, 17	0, 9, 17	0, 2, 11	1, 15, 17	1, 9, 12	1, 2, 10	0, 11, 15	0, 3, 12	8, 10, 12
0, 9, 14	0, 2, 8	1, 14, 17	0, 4, 17	0, 1, 5	0, 11, 12	1, 4, 10	9, 10, 15	0, 9, 11	0, 1, 17	1, 14, 15	0, 4, 10	8, 16, 17

Table 3. Representative of orbits under action of θ_1^2 to construct SRHG(2, 4, 19)

0, 1, 2, 3	0, 1, 3, 5	0, 1, 4, 9	0, 1, 6, 9	0, 1, 7, 18	0, 1, 11, 18	0, 1, 13, 16	0, 1, 15, 18	0, 2, 5, 8
0, 2, 7, 16	0, 2, 9, 14	0, 2, 7, 17	0, 2, 10, 15	0, 2, 7, 18	0, 2, 10, 16	0, 2, 8, 9	0, 2, 11, 17	0, 2, 8, 10
0, 2, 11, 18	0, 2, 8, 11	0, 2, 12, 15	0, 2, 8, 12	0, 2, 12, 16	0, 2, 9, 10	0, 2, 12, 17	0, 2, 9, 13	0, 2, 12, 18
0, 1, 2, 4	0, 1, 3, 6	0, 1, 4, 10	0, 1, 6, 10	0, 1, 8, 9	0, 1, 12, 13	0, 1, 13, 17	0, 1, 16, 17	0, 2, 5, 9
0, 1, 2, 5	0, 1, 3, 8	0, 1, 4, 11	0, 1, 6, 11	0, 1, 8, 10	0, 1, 12, 14	0, 1, 13, 18	0, 1, 16, 18	0, 2, 5, 10
0, 1, 2, 6	0, 1, 3, 9	0, 1, 5, 8	0, 1, 7, 11	0, 1, 8, 13	0, 1, 12, 15	0, 1, 14, 15	0, 1, 17, 18	0, 2, 5, 11
0, 1, 2, 8	0, 1, 3, 10	0, 1, 5, 9	0, 1, 7, 12	0, 1, 9, 14	0, 1, 12, 16	0, 1, 14, 16	0, 2, 4, 6	0, 2, 5, 12
0, 1, 2, 9	0, 1, 3, 11	0, 1, 5, 10	0, 1, 7, 13	0, 1, 9, 15	0, 1, 12, 17	0, 1, 14, 17	0, 2, 4, 8	0, 2, 7, 11
0, 1, 2, 10	0, 1, 4, 5	0, 1, 5, 11	0, 1, 7, 14	0, 1, 10, 15	0, 1, 12, 18	0, 1, 14, 18	0, 2, 4, 9	0, 2, 7, 12
0, 1, 2, 11	0, 1, 4, 6	0, 1, 5, 12	0, 1, 7, 16	0, 1, 10, 16	0, 1, 13, 14	0, 1, 15, 16	0, 2, 4, 10	0, 2, 7, 13
0, 1, 3, 4	0, 1, 4, 8	0, 1, 6, 8	0, 1, 7, 17	0, 1, 11, 17	0, 1, 13, 15	0, 1, 15, 17	0, 2, 4, 11	0, 2, 7, 14
0, 3, 7, 16	0, 4, 9, 10	0, 5, 12, 14	0, 8, 9, 11	0, 9, 13, 16	0, 11, 14, 15	1, 3, 12, 16	1, 8, 14, 17	0, 4, 9, 11
0, 2, 12, 14	0, 3, 8, 11	0, 4, 10, 18	0, 5, 14, 16	0, 8, 12, 14	0, 9, 15, 16	0, 12, 14, 16	1, 5, 8, 11	1, 8, 16, 17
0, 2, 13, 15	0, 3, 8, 12	0, 4, 11, 14	0, 5, 14, 17	0, 8, 12, 17	0, 9, 15, 17	0, 12, 16, 18	1, 5, 8, 15	1, 8, 16, 18
0, 2, 13, 16	0, 3, 9, 10	0, 4, 11, 16	0, 5, 14, 18	0, 8, 12, 18	0, 9, 16, 17	1, 3, 5, 8	1, 5, 9, 15	1, 8, 17, 18
0, 2, 13, 17	0, 3, 9, 11	0, 4, 11, 17	0, 5, 15, 16	0, 8, 13, 14	0, 9, 17, 18	1, 3, 5, 13	1, 5, 9, 16	1, 9, 10, 14
0, 3, 7, 17	0, 5, 12, 15	0, 8, 9, 12	0, 9, 13, 18	0, 11, 14, 17	1, 3, 13, 14	1, 8, 14, 18	1, 10, 14, 17	1, 10, 15, 15
0, 2, 13, 18	0, 3, 9, 12	0, 4, 11, 18	0, 5, 15, 17	0, 8, 13, 15	0, 10, 11, 12	1, 3, 5, 14	1, 5, 9, 17	1, 9, 10, 15
0, 2, 14, 15	0, 3, 10, 11	0, 4, 16, 17	0, 5, 15, 18	0, 8, 13, 16	0, 10, 11, 13	1, 3, 5, 15	1, 5, 9, 18	1, 9, 10, 16
0, 2, 14, 16	0, 3, 11, 12	0, 4, 16, 18	0, 7, 8, 9	0, 8, 13, 17	0, 10, 11, 14	1, 3, 5, 16	1, 5, 10, 15	1, 9, 10, 17
0, 2, 14, 17	0, 3, 11, 15	0, 4, 17, 18	0, 7, 8, 10	0, 8, 13, 18	0, 10, 11, 16	1, 3, 5, 17	1, 5, 10, 16	1, 9, 10, 18
0, 2, 14, 18	0, 3, 12, 13	0, 5, 7, 11	0, 7, 8, 11	0, 8, 14, 15	0, 10, 11, 17	1, 3, 5, 1, 18	1, 5, 10, 17	1, 9, 11, 12
0, 3, 7, 18	0, 4, 9, 13	0, 5, 13, 16	0, 8, 10, 15	0, 9, 14, 15	0, 11, 16, 17	1, 3, 13, 15	1, 8, 15, 16	0, 3, 8, 9
0, 2, 15, 15	0, 3, 12, 14	0, 5, 7, 12	0, 7, 8, 12	0, 8, 14, 16	0, 10, 11, 18	1, 3, 8, 9	1, 5, 10, 18	1, 9, 11, 13
0, 2, 15, 17	0, 3, 12, 15	0, 5, 7, 13	0, 7, 8, 16	0, 8, 14, 17	0, 10, 12, 13	1, 3, 8, 10	1, 5, 11, 15	1, 9, 11, 14
0, 2, 15, 18	0, 3, 13, 14	0, 5, 7, 14	0, 7, 8, 17	0, 8, 14, 18	0, 10, 12, 14	1, 3, 8, 12	1, 8, 9, 10	1, 9, 11, 15
0, 2, 16, 17	0, 3, 13, 15	0, 5, 7, 16	0, 7, 8, 18	0, 8, 15, 16	0, 10, 12, 15	1, 3, 9, 14	1, 8, 9, 13	1, 9, 11, 16
0, 2, 16, 18	0, 3, 13, 16	0, 5, 7, 17	0, 7, 9, 10	0, 8, 15, 17	0, 10, 12, 16	1, 3, 9, 15	1, 8, 10, 12	1, 9, 11, 17
1, 10, 16, 18	0, 4, 10, 16	0, 5, 13, 17	0, 8, 11, 14	0, 9, 14, 16	0, 11, 16, 18	1, 5, 8, 9	1, 8, 15, 17	1, 11, 14, 16
0, 2, 17, 18	0, 3, 13, 17	0, 5, 7, 18	0, 7, 9, 11	0, 8, 17, 18	0, 10, 12, 17	1, 3, 9, 16	1, 8, 10, 18	1, 9, 11, 18
0, 3, 4, 8	0, 3, 13, 18	0, 5, 8, 10	0, 7, 9, 12	0, 8, 16, 17	0, 10, 12, 18	1, 3, 9, 17	1, 8, 11, 12	1, 9, 12, 13
0, 3, 4, 9	0, 3, 14, 15	0, 5, 8, 11	0, 7, 9, 13	0, 8, 16, 18	0, 10, 13, 14	1, 3, 9, 18	1, 8, 11, 14	1, 9, 12, 14
0, 3, 4, 10	0, 3, 14, 16	0, 5, 8, 16	0, 7, 9, 17	0, 8, 17, 18	0, 10, 13, 17	1, 3, 10, 12	1, 8, 11, 15	1, 9, 12, 16
0, 3, 8, 10	0, 4, 10, 17	0, 5, 13, 18	0, 8, 11, 15	0, 9, 14, 18	0, 12, 13, 17	1, 5, 8, 10	1, 8, 15, 18	8, 9, 14, 16
0, 3, 4, 16	0, 3, 14, 17	0, 5, 8, 17	0, 7, 9, 18	0, 9, 10, 15	0, 10, 13, 18	1, 3, 10, 13	1, 8, 11, 16	1, 9, 12, 17
0, 3, 4, 17	0, 3, 14, 18	0, 5, 8, 18	0, 7, 10, 11	0, 9, 10, 16	0, 10, 14, 17	1, 3, 10, 16	1, 8, 11, 17	1, 9, 12, 18
0, 3, 4, 18	0, 3, 15, 16	0, 5, 9, 15	0, 7, 10, 12	0, 9, 10, 17	0, 10, 14, 18	1, 3, 10, 17	1, 8, 12, 13	1, 9, 13, 14

Table 3. Continue

0, 3, 5, 8	0, 3, 15, 17	0, 5, 9, 16	0, 7, 10, 13	0, 9, 11, 13	0, 10, 15, 18	1, 3, 10, 18	1, 8, 12, 16	1, 9, 13, 16
0, 3, 5, 9	0, 3, 15, 18	0, 5, 9, 17	0, 7, 10, 14	0, 9, 11, 14	0, 10, 17, 18	1, 3, 11, 12	1, 8, 12, 17	1, 10, 11, 12
1, 11, 15, 16	8, 9, 14, 17	8, 9, 14, 18	8, 9, 15, 16	8, 9, 15, 17	8, 9, 15, 18	8, 9, 16, 17	8, 9, 16, 18	8, 10, 12, 15
0, 3, 5, 10	0, 3, 16, 17	0, 5, 9, 18	0, 7, 10, 15	0, 9, 11, 16	0, 11, 12, 13	1, 3, 11, 13	1, 8, 12, 18	1, 10, 11, 13
0, 3, 5, 16	0, 3, 16, 18	0, 5, 10, 13	0, 7, 10, 16	0, 9, 11, 18	0, 11, 12, 14	1, 3, 11, 14	1, 8, 13, 14	1, 10, 11, 14
0, 3, 5, 17	0, 3, 17, 18	0, 5, 10, 14	0, 7, 11, 15	0, 9, 12, 15	0, 11, 12, 16	1, 3, 11, 15	1, 8, 13, 15	1, 10, 11, 15
0, 3, 5, 18	0, 4, 8, 9	0, 5, 10, 15	0, 7, 13, 15	0, 9, 12, 16	0, 11, 12, 17	1, 3, 11, 17	1, 8, 13, 16	1, 10, 11, 18
0, 3, 7, 11	0, 4, 8, 10	0, 5, 11, 13	0, 7, 14, 15	0, 9, 12, 17	0, 11, 12, 18	1, 3, 11, 18	1, 8, 13, 17	1, 10, 12, 13
0, 3, 7, 12	0, 4, 8, 11	0, 5, 11, 14	0, 7, 15, 17	0, 9, 12, 18	0, 11, 13, 14	1, 3, 12, 13	1, 8, 13, 18	1, 10, 12, 17
0, 3, 7, 13	0, 4, 8, 12	0, 5, 11, 15	0, 7, 15, 18	0, 9, 13, 14	0, 11, 13, 15	1, 3, 12, 14	1, 8, 14, 15	1, 10, 13, 14
0, 3, 7, 14	0, 4, 8, 13	0, 5, 12, 13	0, 8, 9, 10	0, 9, 13, 15	0, 11, 13, 16	1, 3, 12, 15	1, 8, 14, 16	1, 10, 13, 15
8, 11, 16, 17	8, 11, 17, 18	8, 13, 16, 18	8, 15, 17, 18	8, 16, 17, 18	9, 11, 13, 15	9, 11, 13, 17	9, 11, 16, 18	9, 11, 17, 18
9, 13, 16, 18	9, 13, 17, 18	1, 11, 16, 17	1, 11, 16, 18	1, 12, 14, 17	1, 12, 14, 18	1, 12, 15, 16	1, 12, 15, 18	1, 12, 17, 18
1, 13, 14, 16	1, 13, 14, 18	1, 13, 15, 17	1, 13, 15, 18	1, 13, 16, 17	1, 13, 17, 18	1, 14, 15, 16	1, 14, 15, 17	1, 14, 15, 18
1, 14, 16, 18	1, 14, 17, 18	1, 15, 16, 17	1, 15, 16, 18	1, 15, 17, 18	1, 16, 17, 18	8, 9, 10, 11	8, 9, 10, 12	8, 9, 11, 16
8, 9, 11, 18	8, 9, 12, 14	8, 9, 12, 15	8, 9, 12, 16	8, 9, 13, 14	8, 9, 13, 15	8, 9, 13, 17	8, 9, 13, 18	8, 10, 12, 16
9, 13, 16, 17	1, 13, 14, 15	1, 14, 16, 17	8, 9, 11, 17	8, 11, 15, 18	8, 11, 15, 17	8, 10, 13, 17	8, 10, 16, 17	8, 11, 12, 16
8, 11, 12, 15	8, 11, 13, 16	8, 11, 13, 18	8, 10, 12, 18	8, 10, 13, 15				

Table 4. Representative of orbits under action of θ_2 to construct RHG(2, 3, 18)

0, 1, 3	0, 2, 4	0, 3, 11	0, 9, 14	0, 12, 14	0, 15, 16	8, 10, 12	0, 1, 5	0, 2, 9	0, 3, 13	0, 9, 16	0, 12, 16	0, 15, 17
8, 10, 13	0, 1, 8	0, 2, 10	0, 3, 14	0, 9, 17	0, 12, 17	0, 16, 17	8, 12, 17	0, 1, 9	0, 2, 11	0, 3, 15	0, 10, 16	0, 13, 14
8, 9, 11	0, 1, 11	0, 2, 15	0, 4, 9	0, 11, 14	0, 13, 17	8, 9, 12	0, 1, 12	0, 3, 8	0, 8, 14	0, 11, 16	0, 14, 15	8, 9, 13
8, 9, 10	8, 16, 17	0, 1, 10	0, 2, 12	0, 4, 8	0, 10, 17	0, 13, 16	0, 1, 13	0, 3, 10	0, 8, 17	0, 11, 17	0, 14, 16	8, 9, 14

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