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ON THE NUMBER OF MUTUALLY DISJOINT CYCLIC DESIGNS

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Abstract. We denote by $LS[N](t,k,v)$ a large set of t-(v,k,λ) designs of size N, which is a partition of all k-subsets of a v-set into N disjoint t-(v,k,λ) designs and $N=\binom{v-t}{k-t}/\lambda$. We use the notation $N(t,v,k,\lambda)$ as the maximum possible number of mutually disjoint cyclic t-(v,k,λ)designs. In this paper we give some new bounds for $N(2,29,4,3)$ and $N(2,31,4,2)$. Consequently we present new large sets $LS[9](2,4,29), LS[13](2,4,29)$ and $LS[7](2,4,31)$, where their existences were already known.

1. Introduction

Let $D=\{B_1,B_2,\cdots,B_b\}$ be a family of k-subsets (called blocks) of a point set $X=\{1,2,\cdots,v\}$. The set D is called a t-(v,k,λ) design if every t-subset of X is contained in exactly λ blocks of D. The design D is called simple if it has no repeated blocks. The number b of blocks is called the size of the design. When D consists of all $\binom{v}{k}$ possible k-subsets of X, then D is called the complete design and is denoted by $P_k(X)$.

A large set of t-(v,k,λ) designs is a partition of $P_k(X)$ into t-(v,k,λ) designs, and is denoted by $LS[N](t,k,v)$, where $N=\binom{v-t}{k-t}/\lambda$ is the number of parts in the partition [3]. A necessary condition for the existence of an $LS[N](t,k,v)$ is that

$$N \ \mid \ \binom{v-i}{k-i}, \text{for all } i \text{ with } 0\le i\le t.$$

Let S_X be the group of all permutations on X and $\pi\in S_X$. If π acts on D, then we obtain an isomorphic copy of the design, which is denoted by D^π. An element $\pi\in S_X$ is said to be an automorphism of the design if and only if $D^\pi=D$. The set of all automorphisms of D forms a subgroup of S_X and is

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called the automorphism group of the design denoted by $Aut(D)$. If G is a subgroup of $Aut(D)$, we say that D is G-invariant. The design D is said to be cyclic if it has an automorphism π which is a cycle of order v on the point set X. If D is cyclic it is clear that we can regard the points of D as the elements of the additive cyclic group Z_v, in such a way that π is simply the cycle $x \longmapsto x+1$ [1, 3].

Let G be a subgroup of S_X and let $T_1, T_2, \ldots, T_s$ and $K_1, K_2, \ldots, K_r$ (for positive integers r and s) be the orbits of $P_t(X)$ and $P_k(X)$ under the action of G, respectively. Then for a fixed $T \in T_i$, the number of $K \in K_j$ with $T \subseteq K$ is independent of the choice of T and K; we denote this number by a_{ij}. The Kramer-Mesner matrix is the $s \times r$ matrix $A^v_{t,k}(G)$ whose (i,j)-th entry is a_{ij} where T is any representative in T_i. The following theorem, due to Kramer and Mesner [5] gives a method to find G-invariant designs.

Theorem 1.1. *There exists a G-invariant t-(v,k,λ) design β if and only if there exists a vector $u \in \{0,1\}^r$ satisfying the equation $A^v_{t,k}(G)u = \lambda j$, where j is the s-dimensional all one vector.*

Remark 1.2. *One may also adapt the same method for finding large sets. We pick up one from the set of solutions of u and remove the corresponding columns from $A^v_{t,k}(G)$. The resulting matrix $A'^v_{t,k}(G)$ is used in a similar way to find designs via the equation $A'^v_{t,k}(G)u' = \lambda j$. We repeat the procedure until all the orbits on k-subsets are used* [4].

2. Main Results

Let $N(t,v,k,\lambda)$ be the maximum possible number of mutually disjoint cyclic t-(v,k,λ) designs. In what follows we give some new bounds for $N(2,29,4,3)$ and $N(2,31,4,2)$ [1]. First note that:

Remark 2.1. *The converse of Remark 1.2 may also be taken as a method to find designs. Consider Theorem 1.1 and Remark 1.2 above and let B be a matrix of size $s \times q$, where $q \le r$ and all q columns of B are chosen arbitrary from r columns of $A^v_{t,k}(G)$, then any solution of matrix equation $Bv = \lambda j$ yields easily to a solution u of Kramer-Mesner Matrix equation $A^v_{t,k}(G)u = \lambda j$. Just let u be the zero-one vector each of its entry correspond to each column of $A^v_{t,k}(G)$ such that for all columns in B the vector u has the same entry as v and corresponding for the other columns has zero entry.*

Making use of Remark 2.1 we can give a new bound for $N(2,29,4,3)$. In [2], we obtained the bounds $78 \le N(2,29,4,3) \le 117$. In the following theorem these bounds are improved and a new large set compared with the large set given in [2] is presented:

Theorem 2.2. $110 \le N(2,29,4,3) \le 117$.

Proof. Let $X = \{0,1,\ldots,9,a,b,\ldots,s\}$ and let G be the permutation group of order 29 generated by the cycle $(0\ \ 1\ \ \ldots\ \ 9\ \ a\ \ b\ \ \ldots\ \ s)$. In this case $A^{29}_{2,4}(G)$ is of size 14×819. Via a computer programming we could find 110 disjoint binary solutions for the Kramer-Mesner equation $A^{29}_{2,4}(G)u =$

$3j$. The method is based on the following analysis. The matrix $A^{29}_{2,4}(G)$ consists of two equal sub-matrices B_1, B_2, each of size 14×364 and a sub-matrix of size 14×91. By Remark 2.1, the matrix equation built with each of these two matrices B_1 and B_2 gives 39 designs with $\lambda = 3$. So there are 78 solutions for the equation $A^{29}_{2,4}(G)u = 3j$. Deleting the corresponding columns of these 78 solutions in $A^{29}_{2,4}(G)$ gives a matrix of size 14×273 where again Remark 2.1 guide us to find 32 another designs. Now we have 110 number of 2-$(29, 4, 3)$ designs, which are all distinct, since the corresponding columns in $A^{29}_{2,4}(G)$ are distinct. In Table 1 we demonstrate all 110 starter blocks of these designs in columns titled D_i, $1 \leq i \leq 110$. This completes the proof. □

With the above lower bound, we can construct two large sets $LS[13](2, 4, 29)$ and $LS[9](2, 4, 29)$.The existence of these two large sets were already established in [6] and [2], respectively. However our construction differs from their.

Corollary 2.3. *There exists an* $LS[13](2, 4, 29)$.

Proof. Any 108 of the 110 designs given in the proof of Theorem 2.2 can be partitioned into 12 sets $\beta_i, i = 1, 2, \cdots, 12$, each consisting of the blocks of nine cyclic 2-$(29, 4, 3)$ designs. So $\beta_i, i = 1, 2, \cdots, 12$ is a cyclic 2-$(29, 4, 27)$ design. Since $P_4(X)$ is the trivial cyclic 2-$(29, 4, 351)$ design, we have $P_4(X) \setminus \bigcup_{i=1}^{12} \beta_i$, called β_{13}, which is a cyclic 2-$(29, 4, 27)$ design. Now the set $\{\beta_1, \ldots, \beta_{13}\}$ gives the result. □

Corollary 2.4. *There exists an* $LS[9](2, 4, 29)$.

Proof. Any 104 of the 110 designs given in the proof of Theorem 2.2 can be partitioned into 8 sets $\beta_i, i = 1, 2, \cdots, 8$ each of which being a cyclic 2-$(29, 4, 39)$ design. Also $P_4(X) \setminus \bigcup_{i=1}^{8} \beta_i$ called β_9, is a cyclic 2-$(29, 4, 39)$ design. Now the set $\{\beta_1, \ldots, \beta_9\}$ gives the result. □

Theorem 2.5. $174 \leq N(2, 31, 4, 2) \leq 203$.

Proof. Let G be the cyclic group Z_{31}. In this case $A^{31}_{2,4}(G)$ is of size 15×1015. Via a computer programming we could find 174 disjoint binary solutions for the Kramer-Mesner equation $A^{31}_{2,4}(G)u = 2j$. Each of these solutions correspond to a 2-$(31, 4, 2)$ design. In Table 2 we demonstrate all 174 starter blocks of these designs in columns titled D_i, $1 \leq i \leq 174$. This completes the proof. □

Making use of the above lower bound we can construct $LS[7](2, 4, 31)$. The existence of this large set was already established in [6]. However our construction differs from its.

Corollary 2.6. *There exists an* $LS[7](2, 4, 31)$.

Proof. The 174 designs given in the proof of Theorem 2.5 can be partitioned into 6 sets $\beta_i, i = 1, 2, \cdots, 6$ each of which is a cyclic 2-$(31, 4, 58)$ design. Also $P_4(X) \setminus \bigcup_{i=1}^{6} \beta_i$, denoted β_7 is a cyclic 2-$(31, 4, 58)$ design. Now the set $\{\beta_1, dots, \beta_7\}$ gives the result. □

Remark 2.7. *Concerning the three large sets, given above, one question that naturally arises is that whether there exists large sets with smaller* λ *(and consequently with finer partition of the complete design). This is a new challengeable question we would like to be considered at this step.*

Table 1. Starter blocks of 110 distinct 2-(29,4,3)designs

D_{1}	D_{2}	D_{3}	D_{4}	D_{5}	D_{6}	D_{7}	D_{8}	D_{9}	D_{10}	D_{11}	D_{12}	D_{13}
0124	0125	0126	0127	0128	0129	012a	012b	012c	012d	012e	012f	013f
0139	0138	013a	0137	013b	0135	0136	013c	013d	013e	013j	013k	013l
03cj	02ci	025g	02ai	025e	039j	02bf	025f	025b	027m	027a	0269	014a
04ci	03ci	03bk	03ch	03cg	03di	037j	037m	038m	037o	038c	038g	027l
04di	04bj	04bh	03dh	04af	04bi	04ai	048d	048g	038n	048f	049n	049h
05bl	04dj	04cj	04bk	04am	04ch	05ci	05bh	049f	04ak	05bk	04bl	04ah
05cj	05cl	05dj	05bj	05dk	06ck	05ck	06dl	06dk	04al	06cj	05bm	05bi
D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}	D_{26}
013g	013h	013i	013p	013q	0146	0147	0148	014g	014h	014k	014m	014o
013m	013n	013o	0149	014b	014d	014e	014f	014i	014j	014l	014n	0157
015c	014c	016d	01ah	016f	017e	015d	016g	016b	015n	015b	015h	01ai
028e	028m	028f	02af	02ag	027h	028h	027c	027b	027n	027e	029f	029h
039l	04aj	03ak	02bh	02bi	028j	029k	028k	028g	029e	027g	02aj	02dh
049j	04bg	048i	03bi	04ck	03bj	02ah	02bj	029l	02ak	028i	02bg	038i
049m	05ai	049k	04ag	05ah	049i	05ag	039g	039m	039h	03bh	038e	03ag
D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}
014p	0158	0159	015f	015j	015k	015o	0169	016c	016h	016j	016m	016n
015e	015a	015g	015i	015l	015m	0168	016a	016e	016k	016l	017b	018a
017d	01ce	017j	017m	016i	018k	018l	018e	018h	0179	018b	019b	01be
028d	026i	027f	024c	028b	026c	02cf	024f	024e	025l	024g	025j	025d
029j	028l	029c	029i	029g	027k	02cg	02ch	025k	029d	026m	026h	026k
02be	039i	02ad	036c	02ae	02dg	037i	037k	037b	037h	038k	037g	03af
03ai	03ah	039f	038j	036d	036e	039k	03al	03aj	03bf	03ae	038h	048h
D_{40}	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}	D_{49}	D_{50}	D_{51}	D_{52}
012r	012q	012p	012o	012n	012m	012l	012k	012j	012i	012h	012g	01fr
01lr	01mr	01kr	01nr	01jr	01pr	01or	01ir	01hr	01gr	01br	01ar	019r
03dk	02dj	02fq	02dl	02hq	03dn	02gk	02gq	02kq	029o	02lo	02mp	01kq
04fl	03ek	03cl	03fk	03gk	03ej	03dp	03ap	03ao	038p	03ko	03go	02ao
04fk	04em	04gm	03fj	04in	04fm	04fn	048o	048l	039o	048m	04ao	04go
05dn	04ek	04el	04dm	04bn	04gl	05gm	05hn	04io	04dn	05en	04cm	04gn
05fm	05dm	05fl	05fn	05el	06cl	05em	06em	06fm	04cn	06cm	05cn	05gn
D_{53}	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}	D_{61}	D_{62}	D_{63}	D_{64}	D_{65}
01er	01dr	01cr	015r	014r	01oq	01nq	01mq	01eq	01dq	01aq	018q	016q
018r	017r	016r	01lq	01jq	01hq	01gq	01fq	01cq	01bq	019q	017q	01np
01ip	01iq	01ho	01dk	01fo	01gn	01hp	01eo	01jo	017p	01jp	01dp	01ck
02hn	029n	02gn	02gl	02fl	02eo	02en	02jo	02ko	028o	02ho	02gm	02em
03bn	04en	03cm	02ek	02dk	02cn	02bm	02bn	02fn	02hm	02fo	02cl	02ei
04eo	04hm	048j	03el	04dl	03dl	02el	02ck	02am	02bl	02dn	02fk	03eo
04bo	05al	04do	04hn	05am	04fo	05an	03gn	03an	03fn	03fl	03io	03gm
D_{66}	D_{67}	D_{68}	D_{69}	D_{70}	D_{71}	D_{72}	D_{73}	D_{74}	D_{75}	D_{76}	D_{77}	D_{78}
015q	01mp	01lp	01fp	01bp	01ap	016p	01lo	01io	01do	01bo	018o	017o
01gp	01kp	01ep	01cp	019p	018p	01mo	01ko	01go	01ao	019o	01jn	01km
01hn	01gi	01bn	018n	01co	01am	019m	01gm	01dm	01ln	01jm	01jl	01gj
02in	02dp	02go	024l	02kn	02jp	02gj	024i	024j	02aq	024h	02cq	02iq
02cm	02an	02jm	02dm	02fm	02bo	02fj	02ej	02bq	02im	029p	02ep	02bp
02hk	03en	02il	036n	02hl	02fi	03ep	03cp	03lp	03fp	03co	03gp	03hm
03em	03fm	03hn	03do	036m	036l	03cn	03bm	03dm	03hl	03im	03fo	048k
D_{79}	D_{80}	D_{81}	D_{82}	D_{83}	D_{84}	D_{85}	D_{86}	D_{87}	D_{88}	D_{89}	D_{90}	D_{91}
0123	0134	013r	0145	014q	0156	0167	016o	0178	017i	017l	017n	0189
027d	017h	015p	017a	017c	017f	018i	018f	018j	017k	018c	018d	019e
039n	02bd	01cg	02al	017g	027j	026l	019c	025i	01ad	01ae	019j	025m
03am	038l	02cj	02eg	028a	028c	029b	024d	026f	0246	025a	0258	02cp
04cl	048e	03bl	038f	02di	02bk	038d	026g	02jn	037f	026j	02ce	02do
04ei	05cm	06dm	04dk	03ck	037l	03eh	03ad	03cf	03bo	027i	03ei	036h
05di	05ek	06ek	05bn	04ej	03dg	04dh	049l	049e	05ej	036f	049d	04ae

Table 1. (continue)

D_{92}	D_{93}	D_{94}	D_{95}	D_{96}	D_{97}	D_{98}	D_{99}	D_{100}	D_{101}	D_{102}	D_{103}	D_{104}
018g	018m	019a	019f	019i	019l	019n	01ag	01ak	01al	01an	01bc	01bf
019d	019k	019g	019h	01af	01ac	01ab	01aj	01ch	01bg	01bm	01df	01ci
01bi	01ei	027p	01bh	01bd	01eg	026a	01in	01dg	01hm	01eh	026n	01dl
025c	025h	02co	0257	025q	026p	02eh	0247	0248	0268	024q	029m	0279
025p	026o	02dq	02jq	026e	037a	02io	02hp	026b	02eq	02ap	037d	02gp
02fp	026q	036i	039d	037p	03ho	037c	03gl	036b	037e	03in	038b	036a
D_{105}	D_{106}	D_{107}	D_{108}	D_{109}	D_{110}							
01bj	01bk	01bl	01cd	01cl	01cm							
01cf	01di	01dh	01cj	01fh	01de							
01en	01fl	01fm	024n	01hl	0249							
024a	024p	026d	0259	025n	028p							
02nq	027o	02kp	036j	025o	037n							
049g	036g	02lq	049o	03ip	03be							
04ho	03bp	036k	06el	04an	05aj							

Table 2. Starter blocks of 174 distinct 2-(31,4,2) designs

D_{1}	D_{2}	D_{3}	D_{4}	D_{5}	D_{6}	D_{7}	D_{8}	D_{9}	D_{10}	D_{11}	D_{12}	D_{13}
0124	0125	0126	0127	0128	0129	012a	012b	012c	012d	012e	012f	012g
038i	026h	025g	025d	025c	025b	025e	0259	0258	025a	025m	025o	025r
04dk	03cl	03dm	03cj	03ei	03dj	037f	03di	04ci	03ae	037r	038n	03an
05cm	05ck	04bj	04dl	04cm	04ei	04fl	04cj	04fm	04hn	05dl	04am	04cl
06ck	06dn	06eo	04ek	05ek	05go	05bi	06ek	05el	06fm	06cl	04ao	05bo
D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}	D_{26}
012h	012i	012j	012k	012l	012m	012n	012o	012p	012q	012r	012s	012t
0269	025p	025q	025l	025f	025h	025i	025j	025k	026d	026i	026j	038j
03al	037n	039i	037d	037c	038c	037e	037g	037j	03bl	03bo	03bk	04bl
049n	04dp	04bp	049o	04bh	04io	049j	04ak	04al	03fj	03ck	05cl	05dj
05bn	05eo	04ck	06en	06em	06dl	06cn	05dp	05dm	05em	05fm	06go	06dm
D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}
01ek	01ep	01eq	01fi	01fj	01fk	01fl	01fp	01fr	01fs	01gl	01gr	01gs
01go	01ko	01fq	01kp	01jn	01mt	01ms	01nr	01js	01lo	01iq	01lt	01mo
024m	02hl	029b	02an	0279	02lr	027j	02co	027m	028r	024n	02jq	02jn
036a	02ns	03ad	02mr	036e	03bi	029d	02di	028p	02fm	039c	03cp	03kp
05aj	03gp	048g	03cr	05fp	04iq	03iq	036n	03do	05am	04ho	04hp	05ip
D_{40}	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}	D_{49}	D_{50}	D_{51}	D_{52}
0134	0135	0136	0137	0138	0139	013a	013b	013g	013h	013i	013j	013l
028i	016g	015g	013e	013k	013c	013d	013f	013p	013q	013o	0149	014a
04em	03em	02aj	04bn	04ch	04ah	048f	049d	04cq	04an	049g	029f	028f
05bk	04dn	04en	05en	04fp	04ej	05gm	05cp	04ep	04bm	04cp	04eo	04go
05cj	06dk	06do	05fl	06fo	05dk	05hn	06el	05cn	05fo	05bm	05bj	05ej
D_{53}	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}	D_{61}	D_{62}	D_{63}	D_{64}	D_{65}
013m	013n	013r	013s	013t	0145	0146	0147	0148	014c	014f	014g	014h
014d	014b	014e	015i	018i	027i	015f	015e	016e	014i	014m	0157	014n
028g	028h	02bj	02bl	04bk	02em	02dk	02ai	029k	028l	028k	029j	027o
04bi	04gl	05do	05hp	04in	039l	03am	02cj	02ch	029o	028o	039k	02dp
05ag	05ci	06fl	07fm	06cm	06gn	06fn	05gp	039j	05ap	05ah	05dn	05bl
D_{66}	D_{67}	D_{68}	D_{69}	D_{70}	D_{71}	D_{72}	D_{73}	D_{74}	D_{75}	D_{76}	D_{77}	D_{78}
013m	013n	013r	013s	013t	0145	0146	0147	0148	014c	014f	014g	014h
014d	014b	014e	015i	018i	027i	015f	015e	016e	014i	014m	0157	014n
028g	028h	02bj	02bl	04bk	02em	02dk	02ai	029k	028l	028k	029j	027o
04bi	04gl	05do	05hp	04in	039l	03am	02cj	02ch	029o	028o	039k	02dp
05ag	05ci	06fl	07fm	06cm	06gn	06fn	05gp	039j	05ap	05ah	05dn	05bl
D_{79}	D_{80}	D_{81}	D_{82}	D_{83}	D_{84}	D_{85}	D_{86}	D_{87}	D_{88}	D_{89}	D_{90}	D_{91}
015m	015p	015q	015r	015s	015t	0167	0168	0169	016b	016d	016h	016j
015o	016n	017f	017e	016k	017h	024g	016l	016a	018b	016m	016s	017a
028j	024h	02be	028m	02ak	02fl	038m	02dg	02fh	02er	024b	029l	02ae
02bi	03dn	02dl	02al	02ho	03bm	03dl	03co	03el	02go	03bj	02fn	02bg
036g	03fm	03ir	03fi	03ci	049h	04fo	048h	04co	03cg	03dh	037k	03br

Table 2. (continue)

D_{92}	D_{93}	D_{94}	D_{95}	D_{96}	D_{97}	D_{98}	D_{99}	D_{100}	D_{101}	D_{102}	D_{103}	D_{104}
016o	016p	016q	016r	016t	0178	0179	017b	017c	017d	017j	017l	017n
017t	018a	018h	018c	017k	024j	017i	017g	018n	019b	018t	017r	017s
02dn	02bf	02ac	02ao	02hm	038l	02is	02ce	02fs	027h	02im	024i	02df
03ch	03fq	03er	02hk	03aj	03ep	03bf	038b	02jm	03gk	038h	038g	038k
04fj	03hl	03im	03gm	048i	04gq	049l	04di	04ae	03mq	04af	03cm	04eq
D_{105}	D_{106}	D_{107}	D_{108}	D_{109}	D_{110}	D_{111}	D_{112}	D_{113}	D_{114}	D_{115}	D_{116}	D_{117}
017o	017p	017q	0189	018d	018e	018f	018g	018j	018k	018m	018o	018p
019c	019l	019m	024l	018s	019s	01ac	01at	018l	019e	018r	01bd	01ad
024e	02cs	025n	03dg	02bh	026k	026l	02ls	026s	026g	02dr	02dh	024f
03gl	02hs	02eh	04bq	02gl	029c	036b	04dh	02gj	027b	02ds	036c	03cq
049f	048m	04bf	05bh	039d	05fk	04hm	05bp	049p	036l	03fp	049e	04kp
D_{118}	D_{119}	D_{120}	D_{121}	D_{122}	D_{123}	D_{124}	D_{125}	D_{126}	D_{127}	D_{128}	D_{129}	D_{130}
018q	019a	019f	019g	019h	019i	019j	019n	019o	019p	019q	019r	019t
019d	024k	019k	01bj	01be	01bh	01af	01bf	01bs	01as	01bi	01bo	01ae
026m	037l	027s	026q	026c	027r	028s	027c	027k	027d	02cr	027l	027e
02gs	038f	02ei	02es	02fq	02as	02fr	02kr	02js	02cg	02ks	02gr	03bg
03ak	05jp	03lr	03lp	03mr	03fr	03kr	036i	04ag	03eq	037m	036f	04jp
D_{131}	D_{132}	D_{133}	D_{134}	D_{135}	D_{136}	D_{137}	D_{138}	D_{139}	D_{140}	D_{141}	D_{142}	D_{143}
01ab	01ag	01ah	01ai	01aj	01ak	01al	01am	01an	01ao	01ap	01aq	01ar
026e	01cj	01bt	01bq	01ch	01dh	01ft	01io	01cf	01cr	01cn	01ci	01ce
02iq	0257	027p	029s	0268	02ip	028q	02gk	02hr	026n	025s	0249	02kp
037p	03bp	039e	02jr	037o	02os	037q	02ps	02kq	02dq	02fj	03dr	03ap
03ej	048l	04cg	037i	03in	038e	04dj	04bg	038r	036m	04ip	03fn	03jr
D_{144}	D_{145}	D_{146}	D_{147}	D_{148}	D_{149}	D_{150}	D_{151}	D_{152}	D_{153}	D_{154}	D_{155}	D_{156}
01bc	01bg	01bk	01bn	01bp	01br	01cd	01cg	01ck	01co	01cq	01ct	01de
029q	01cp	01er	01ds	01cs	01dj	024q	01em	01hm	01hr	01dg	01el	026r
02ag	024c	02hn	026o	02eo	024o	03bh	029r	0247	024d	027a	027n	027g
036j	039r	02or	02gq	02ir	038o	03go	02cq	039g	038p	029m	039f	03aq
048d	03hq	03jq	03kq	038q	039h	04aq	036q	04ai	03ag	048e	048q	03cn
D_{157}	D_{158}	D_{159}	D_{160}	D_{161}	D_{162}	D_{163}	D_{164}	D_{165}	D_{166}	D_{167}	D_{168}	D_{169}
01df	01di	01dk	01dl	01dm	01dn	01do	01dp	01dq	01dr	01dt	01ei	01ej
01en	01gn	01hs	01es	01oq	01lq	01eh	01nt	01fm	01eo	01in	01ot	01gq
028c	02lp	028a	02aq	02ar	02eg	026b	02af	024a	02br	026p	02cn	02bp
037a	02ms	03io	02mq	03fk	037b	02nq	037h	03be	02mp	03ao	039p	02lo
05bg	03ko	049q	03jp	03gr	03ip	04aj	049k	04jq	03af	04fq	04jo	03nr
D_{170}	D_{171}	D_{172}	D_{173}	D_{174}								
01hj	01hl	01hn	01ip	01jo								
01nq	01ns	01it	01ks	01kt								
02do	02ad	02lq	02bs	02fp								
03jo	02hq	03dp	02in	03hr								
048p	06cj	048o	04gp	04kq								

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