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DEGREE DISTANCE AND GUTMAN INDEX OF CORONA PRODUCT OF GRAPHS

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Abstract. In this paper, the degree distance and the Gutman index of the corona product of two graphs are determined. Using the results obtained, the exact degree distance and Gutman index of certain classes of graphs are computed.

1. Introduction

In this paper, all graphs considered are simple, connected and finite. Let $G = (V(G), E(G))$ be a connected graph. For $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u, v)$, is the length of a shortest (u, v)-path in G. The degree of a vertex $w \in V(G)$ is denoted by $d_G(w)$ and we denote $(d_G(w))^2$ by $d_G^2(w)$. Let P_n, C_n and S_n denote the path, the cycle and the star on n vertices, respectively. The number of edges of G is denoted by $\epsilon(G)$. Notation and definitions which are not given here can be found in [2] or [4].

A *topological index* is a real number related to a graph. It does not depend on the labeling or pictorial representation of a graph. The Wiener index $W(G)$ is the first distance based topological index defined as $W(G) = \sum\limits_{\{u,v\} \subseteq V(G)} d_G(u, v) = \frac{1}{2} \sum\limits_{u,v \in V(G)} d_G(u, v)$ with the summation runs over all pairs of vertices of G [20]. The topological indices and graph invariants based on distances between vertices of a graph are widely used for characterizing molecular graphs, establishing relationships between structure and properties of molecules, predicting biological activity of chemical compounds, and making their chemical applications. These indices may be used to derive quantitative structure-property or structure-activity relationships(QSPR/QSAR). The Wiener index is one of the most used topological indices with high correlation with many physical and chemical indices of molecular compounds [20].

The first and second kinds of Zagreb indices were first introduced in [10] (see also [9]). These topological indices are based on degrees. The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ of a graph G are defined as $M_1(G) = \sum\limits_{uv \in E(G)} [d_G(u) + d_G(v)] = \sum\limits_{v \in V(G)} d_G^2(v)$ and $M_2(G) = \sum\limits_{uv \in E(G)} [d_G(u) d_G(v)]$, respectively.

The degree distance was introduced by Dobrynin and Kochetova [5] and Gutman [11] as a weighted version of the Wiener index. The *degree distance* of G, denoted by $DD(G)$,is defined as $DD(G) = \frac{1}{2} \sum\limits_{u,v \in V(G)} d_G(u, v)[d_G(u) + d_G(v)]$ with the summation runs over all pairs of vertices of G. In [11], Gutman showed that if G is a

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tree on n vertices, then $DD(G) = 4W(G) - n(n-1)$. In [11], Gutman defined the *modified Schultz index*, which is now known as the Gutman index. The *Gutman index* of G, denoted by $Gut(G)$,is defined as $Gut(G) = \sum\limits_{\{u,v\}\subseteq V(G)} d_G(u,v)d_G(u)d_G(v) = \frac{1}{2}\sum\limits_{u,v\in V(G)} d_G(u,v)d_G(u)d_G(v)$ with the summation runs over all pairs of vertices of G. If G is a tree on n vertices, then Wiener index and Gutman index are closely related by $Gut(G) = 4W(G) - (2n-1)(n-1)$; see [11].

The *corona product* of the graphs G_1 and G_2, denoted by $G_1 \odot G_2$, is the graph obtained by taking one copy of G_1 and $|V(G_1)|$ disjoint copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in i^{th} copy of G_2 [8]; see Figure 1.

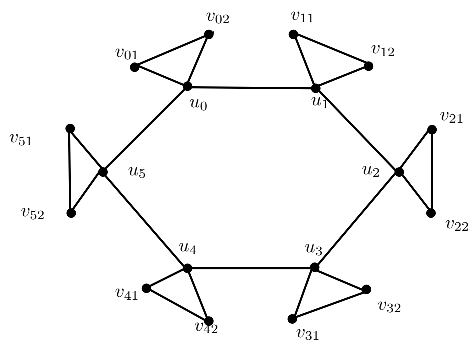

Figure 1: The graph $C_6 \odot K_2$.

It is clear from the definition that corona product of two graph is not commutative. Obviously, $G_1 \odot G_2$ is connected if and only if G_1 is connected.

In [3], [1] and [18], the exact formula for the Wiener, Hyper Wiener, PI, vertex PI, edge Szeged and edge-vertex Szeged indices of the corona product graphs are obtained. In [21], the Szeged, vertex PI and the first and second Zagreb indices of corona product of graphs are computed. In this paper, the degree distance and the Gutman index of the corona product of two graphs are determined. Using the results obtained, the exact degree distance and Gutman index of the graphs $P_m \odot \overline{K}_n$, $P_m \odot P_n$, $P_m \odot C_n$, $C_m \odot C_n$ and $P_m \odot S_n$ are determined.

2. **Degree Distance of Corona Product of Graphs**

In this section, we compute the degree distance of the corona product $G_1 \odot G_2$ of the graphs G_1 and G_2.

Let $V(G_1) = \{u_0, u_1, \ldots, u_{n_1-1}\}$ and $V(G_2) = \{v_0, v_1, \ldots, v_{n_2-1}\}$. For $0 \le i \le n_1 - 1$, denote by G_2^i the i^{th} copy of G_2 joined to the vertex u_i and let $V(G_2^i) = \left\{v_{i0}, v_{i1}, \ldots, v_{i(n_2-1)}\right\}$.

The following lemmas are followed from the definition of the corona product of two graphs.

Lemma 2.1. *Let G_1 be a connected graph and G_2 be a graph. Then the degree of $w \in V(G_1 \odot G_2)$ is*

$$d_{G_1\odot G_2}(w) = \begin{cases} d_{G_1}(w) + n_2, & \text{if } w \in V(G_1) \\ d_{G_2}(w) + 1, & \text{if } w \in V(G_2^i) \text{ for some } 0 \le i \le n_1 - 1. \end{cases}$$

Lemma 2.2. *Let G_1 and G_2 be arbitrary graphs. Let G_2^i be the i^{th} copy of G_2 in $G_1 \odot G_2$ and let $V(G_2^i) = \{v_{i0}, v_{i1}, \ldots, v_{i(n_2-1)}\}$, $0 \le i \le n_1 - 1$. Then*

$$d_{G_1 \odot G_2}(u_i, u_p) = d_{G_1}(u_i, u_p), \ \textit{if } 0 \le i, p \le n_1 - 1,$$
$$d_{G_1 \odot G_2}(u_i, v_{pq}) = d_{G_1}(u_i, u_p) + 1, \ \textit{if } 0 \le i, p \le n_1 - 1, 0 \le q \le n_2 - 1,$$
$$d_{G_1 \odot G_2}(v_{ij}, v_{pq}) = \begin{cases} d_{G_1}(u_i, u_p) + 2, & \textit{if } i \ne p, \\ 1, & \textit{if } i = p \textit{ and } v_j v_q \in E(G_2), \\ 2, & \textit{if } i = p \textit{ and } v_j v_q \notin E(G_2). \end{cases}$$

Theorem 2.3. *Let G_1 and G_2 be arbitrary graphs. Then $DD(G_1 \odot G_2) = (n_2+1)DD(G_1) + 4(n_2+1)\big[n_2 + \epsilon(G_2)\big]W(G_1) + 2n_1n_2\epsilon(G_1) + 2n_1\big(2n_1n_2 + n_1 - 3\big)\epsilon(G_2) - n_1M_1(G_2) + n_1n_2\big(3n_1n_2 + n_1 - 2\big)$.*

Proof. Let $G = G_1 \odot G_2$. Then

$$\begin{aligned}
DD(G) &= \frac{1}{2} \sum_{u,v \in V(G)} d_G(u,v)\big[d_G(u) + d_G(v)\big] \\
&= \frac{1}{2}\Bigg\{ \sum_{\substack{u_i,\, u_p \in V(G_1) \\ i \ne p}} d_G(u_i, u_p)\big[d_G(u_i) + d_G(u_p)\big] + \sum_{i=0}^{n_1-1} \sum_{\substack{v_{ij},\, v_{iq} \in V(G_2^i) \\ j \ne q}} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big] \\
&\quad + 2\sum_{p=0}^{n_1-1} \sum_{\substack{u_i \in V(G_1), \\ v_{pq} \in V(G_2^p)}} d_G(u_i, v_{pq})\big[d_G(u_i) + d_G(v_{pq})\big] + \sum_{\substack{v_{ij} \in V(G_2^i), \\ v_{pq} \in V(G_2^p) \\ i \ne p}} d_G(v_{ij}, v_{pq})\big[d_G(v_{ij}) + d_G(v_{pq})\big] \Bigg\} \\
&= \frac{1}{2}\Bigg\{ \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} d_G(u_i, u_p)\big[d_G(u_i) + d_G(u_p)\big] + \sum_{i=0}^{n_1-1} \sum_{\substack{j,\, q=0 \\ j \ne q}}^{n_2-1} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big] \\
&\quad + 2\sum_{p=0}^{n_1-1} \sum_{i=0}^{n_1-1} \sum_{q=0}^{n_2-1} d_G(u_i, v_{pq})\big[d_G(u_i) + d_G(v_{pq})\big] + \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} \sum_{j,\, q=0}^{n_2-1} d_G(v_{ij}, v_{pq})\big[d_G(v_{ij}) + d_G(v_{pq})\big] \Bigg\}
\end{aligned}$$

$$= \frac{1}{2}\Big(A_1 + A_2 + A_3 + A_4\Big), \tag{2.1}$$

where A_1-A_4 are the sums of the above terms, in order.
We calculate A_1-A_4 of (2.1) separately.

First we compute A_1.

$$\begin{aligned}
A_1 &= \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} d_G(u_i, u_p)\big[d_G(u_i) + d_G(u_p)\big] \\
&= \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} d_G(u_i, u_p)\Big\{\big[d_{G_1}(u_i) + n_2\big] + \big[d_{G_1}(u_p) + n_2\big]\Big\}, \text{ by Lemmas (2.1) and (2.2),} \\
&= \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} d_G(u_i, u_p)\big[d_{G_1}(u_i) + d_{G_1}(u_p)\big] + 2n_2 \sum_{\substack{i,\, p=0 \\ i \ne p}}^{n_1-1} d_G(u_i, u_p)
\end{aligned}$$

$$= 2DD(G_1) + 4n_2W(G_1), \tag{2.2}$$

by the definitions of degree distance and Wiener index of G_1, respectively.
Next we compute $A_2 = \sum_{i=0}^{n_1-1} \sum_{\substack{j,\, q=0 \\ j \ne q}}^{n_2-1} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big]$. For this, initially we calculate $A_2' =$

$\sum\limits_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big]$.

$$\begin{aligned}
A_2' &= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big] + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} d_G(v_{ij}, v_{iq})\big[d_G(v_{ij}) + d_G(v_{iq})\big]\\
&= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[(d_{G_2}(v_j)+1) + (d_{G_2}(v_q)+1)\Big] + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} 2\Big[(d_{G_2}(v_j)+1) + (d_{G_2}(v_q)+1)\Big]\\
&\qquad \text{by Lemmas (2.1) and (2.2),}\\
&= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big] + 2\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big]\\
&= \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big] + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big]\Bigg\}\\
&\qquad + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big],\ \text{by writing the last term into two terms.}\\
&= \sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big] + \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big]\\
&\qquad + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big]\Bigg\} - \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big]\\
&= 2\sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big] - 2\sum_{v_jv_q\in E(G_2)} \Big[d_{G_2}(v_j) + d_{G_2}(v_q) + 2\Big],\\
&\qquad \text{since both } v_jv_q \text{ and } v_qv_j \text{ are accounted in the last term.}\\
&= 8(n_2-1)\epsilon(G_2) + 8\binom{n_2}{2} - 2M_1(G_2) - 4\epsilon(G_2),\ \text{since} \sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1}\Big[d_{G_2}(v_j) + d_{G_2}(v_q)\Big]\\
&\qquad = 4(n_2-1)\epsilon(G_2) \text{ and } \sum_{v_jv_q\in E(G_2)}\Big[d_{G_2}(v_j) + d_{G_2}(v_q)\Big] = M_1(G_2).\\
&= 8n_2\epsilon(G_2) + 4n_2(n_2-1) - 2M_1(G_2) - 12\epsilon(G_2),\ \text{where } |E(G_2)| = \epsilon(G_2). \qquad (2.3)
\end{aligned}$$

Now using ((2.3)), we get

$$\begin{aligned}
A_2 &= \sum_{i=0}^{n_1-1}\Big\{8n_2\epsilon(G_2) + 4n_2(n_2-1) - 2M_1(G_2) - 12\epsilon(G_2)\Big\}\\
&= n_1\Big\{8n_2\epsilon(G_2) + 4n_2(n_2-1) - 2M_1(G_2) - 12\epsilon(G_2)\Big\}. \qquad (2.4)
\end{aligned}$$

Next we compute $A_3 = 2\sum\limits_{p=0}^{n_1-1}\sum\limits_{i=0}^{n_1-1}\sum\limits_{q=0}^{n_2-1} d_G(u_i, v_{pq})\big[d_G(u_i) + d_G(v_{pq})\big]$. For this first we compute $A_3' = \sum\limits_{i=0}^{n_1-1}\sum\limits_{q=0}^{n_2-1} d_G(u_i, v_{pq})\big[d_G(u_i) + d_G(v_{pq})\big]$.

$$\begin{aligned}
A_3' &= \sum_{i=0}^{n_1-1}\sum_{q=0}^{n_2-1}\Big[d_{G_1}(u_i,u_p)+1\Big]\Big[\big(d_{G_1}(u_i)+n_2\big)+\big(d_{G_2}(v_q)+1\big)\Big], \text{ by Lemmas (2.1) and (2.2).}\\
&= \sum_{i=0}^{n_1-1}\sum_{q=0}^{n_2-1} d_{G_1}(u_i,u_p)\big[d_{G_1}(u_i)+(n_2+1)+d_{G_2}(v_q)\big] + \sum_{i=0}^{n_1-1}\sum_{q=0}^{n_2-1}\Big[d_{G_1}(u_i)+(n_2+1)+d_{G_2}(v_q)\Big]\\
&= n_2\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)d_{G_1}(u_i) + n_2(n_2+1)\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + 2\epsilon(G_2)\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + 2n_2\epsilon(G_1)\\
&\quad + n_1n_2(n_2+1) + 2n_1\epsilon(G_2). \qquad (2.5)
\end{aligned}$$

Using ((2.5)), we get

$$\begin{aligned}
A_3 &= 2\sum_{p=0}^{n_1-1}\Bigg\{n_2\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)d_{G_1}(u_i) + n_2(n_2+1)\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + 2\epsilon(G_2)\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)\\
&\quad + 2n_2\epsilon(G_1) + n_1n_2(n_2+1) + 2n_1\epsilon(G_2)\Bigg\}\\
&= 2n_2DD(G_1) + 4n_2(n_2+1)W(G_1) + 8\epsilon(G_2)W(G_1) + 4n_1n_2\epsilon(G_1) + 2(n_1)^2n_2(n_2+1)\\
&\quad + 4(n_1)^2\epsilon(G_2), \qquad (2.6)
\end{aligned}$$

from the definitions of degree distance and Wiener index of G_1, respectively.
Finally, we compute $A_4 = \sum\limits_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1}\sum\limits_{j,\,q=0}^{n_2-1} d_G(v_{ij}, v_{pq})\big[d_G(v_{ij}) + d_G(v_{pq})\big]$. For this, first we calculate A_4' $= \sum\limits_{j,\,q=0}^{n_2-1} d_G(v_{ij}, v_{pq})\big[d_G(v_{ij}) + d_G(v_{pq})\big]$.

$$\begin{aligned}
A_4' &= \sum_{j,\,q=0}^{n_2-1}\Big[d_{G_1}(u_i,u_p)+2\Big]\Big[\big(d_{G_2}(v_j)+1\big)+\big(d_{G_2}(v_q)+1\big)\Big], \text{ by Lemmas (2.1) and (2.2).}\\
&= \Big[d_{G_1}(u_i,u_p)+2\Big]\sum_{j,\,q=0}^{n_2-1}\Big[d_{G_2}(v_j)+d_{G_2}(v_q)+2\Big]\\
&= \Big[d_{G_1}(u_i,u_p)+2\Big]2\sum_{j,\,q=0}^{n_2-1}\Big(d_{G_2}(v_j)+1\Big)\\
&= \Big[d_{G_1}(u_i,u_p)+2\Big]\Big(4n_2\epsilon(G_2)+2(n_2)^2\Big). \qquad (2.7)
\end{aligned}$$

Using ((2.7)), we get

$$\begin{aligned}
A_4 &= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1}\Bigg\{\Big[d_{G_1}(u_i,u_p)+2\Big]\Big(4n_2\epsilon(G_2)+2(n_2)^2\Big)\Bigg\}\\
&= \Big(4n_2\epsilon(G_2)+2(n_2)^2\Big)\Big[2W(G_1)+2n_1(n_1-1)\Big], \qquad (2.8)
\end{aligned}$$

from the definition of Wiener index of G_1.
Using ((2.2)), ((2.4)), ((2.6)) and ((2.8)) in ((2.1)), we get

$$\begin{aligned} DD(G) = \frac{1}{2}\Big\{ & 2DD(G_1) + 4n_2W(G_1) + 8n_1n_2\epsilon(G_2) + 4n_1n_2(n_2-1) - 2n_1M_1(G_2) \\ & - 12n_1\epsilon(G_2) + 2n_2DD(G_1) + 4n_2(n_2+1)W(G_1) + 8\epsilon(G_2)W(G_1) \\ & + 4n_1n_2\epsilon(G_1) + 2(n_1)^2n_2(n_2+1) + 4(n_1)^2\epsilon(G_2) + \Big(4n_2\epsilon(G_2) \\ & + 2(n_2)^2\Big)\Big[2W(G_1) + 2n_1(n_1-1)\Big]\Big\} \\ = {} & (n_2+1)DD(G_1) + 4(n_2+1)\big[n_2 + \epsilon(G_2)\big]W(G_1) + 2n_1n_2\epsilon(G_1) \\ & + 2n_1\big(2n_1n_2 + n_1 - 3\big)\epsilon(G_2) - n_1M_1(G_2) + n_1n_2\big(3n_1n_2 + n_1 - 2\big). \end{aligned}$$

Corollary 2.4. *If G is a nontrivial connected graph with $|V(G)| = m$, then $DD(G \odot \overline{K}_n) = (n+1)DD(G) + 4n(n+1)W(G) + 2mn\epsilon(G) + mn(3mn + m - 2)$.*

For our future reference we quote the following Lemmas.

Lemma 2.5. [16] *Let P_n and C_n denote the path and the cycle on n vertices, respectively. Then for $n \geq 2$, $W(P_n) = \frac{1}{6}n(n^2-1)$ and for $n \geq 3$,*

$$W(C_n) = \begin{cases} \dfrac{n^3}{8}, & \text{if } n \text{ is even} \\ \dfrac{n(n^2-1)}{8}, & \text{if } n \text{ is odd.} \end{cases}$$

Lemma 2.6. [19, 6] *Let P_n and C_n denote the path and the cycle on n vertices, respectively. Then for $n \geq 2$, $DD(P_n) = \frac{1}{3}n(n-1)(2n-1)$ and for $n \geq 3$,*

$$DD(C_n) = \begin{cases} \dfrac{n^3}{2}, & \text{if } n \text{ is even} \\ \dfrac{n(n^2-1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

Lemma 2.7. [13] *Let P_n and C_n denote the path and the cycle on n vertices, respectively. Then (i) for $n \geq 2$, $M_1(P_n) = 4n - 6$ (ii) for $n \geq 3$, $M_1(C_n) = 4n = M_2(C_n)$ (iii) for $n \geq 3$, $M_2(P_n) = 4(n-2)$ and $M_2(P_1) = 0$, $M_2(P_2) = 1$.*

Let $\{G_i\}_{i=1}^n$ be a set of finite pairwise disjoint graphs with $v_i \in V(G_i)$. The *bridge graph* $B\{G_1, G_2, \ldots, G_n; v_1, v_2, \ldots, v_n\}$ of $\{G_i\}_{i=1}^n$ with respect to the vertices $v_1, v_2, \ldots, v_n$ is the graph obtained from the graphs $G_1, G_2, \ldots, G_n$ by connecting the vertices v_i and v_{i+1} by an edge for all $i = 1, 2, \ldots, n-1$; see Figure 2 [1].

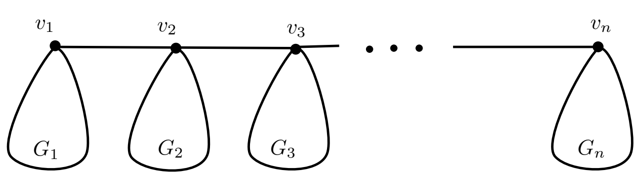

Figure 2: Bridge Graph.

For $m \geq 2$, we define $H_m(G, v) = B\{\underbrace{G, G, \ldots, G}_{m \text{ times}}; \underbrace{v, v, \ldots, v}_{m \text{ times}}\}$, which is the special case of the bridge graph. Clearly, $H_1(G, v) = G$ for any vertex v of G. A few examples are given below.

Example 2.8. *Let P_n be path with vertex set $\{v_1, v_2, \ldots, v_n\}$. Define $B_m = H_m(P_3, v_2)$* (polythene when *$m = 4$;) see Figure 3.*

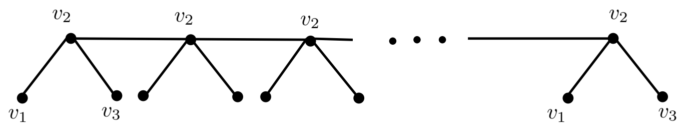

Figure 3: $B_m = H_m(P_3, v_2)$.

Example 2.9. *Define $T_{m,k} = H_m(C_k, v_1)$. For example, $T_{m,3} = H_m(C_3, v_1)$; see Figure 4.*

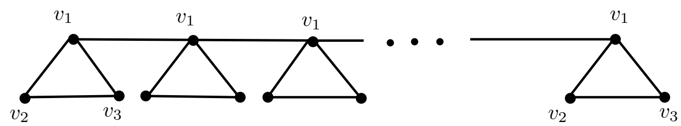

Figure 4: $T_{m,3} = H_m(C_3, v_1)$.

Example 2.10. *Define $J_{m,n+1} = H_m(W_{n+1}, v_1)$, where W_{n+1} is the wheel graph on $n+1$ vertices $v_1, v_2, \ldots, v_n, v_{n+1}\}$ such that degree of v_1 is n and degree of v_i is 3 for $i = 1, 2, \ldots, n$; see Figure 5.*

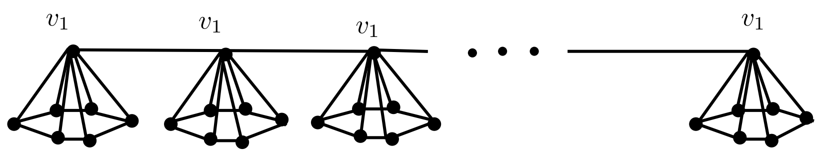

Figure 5: $J_{m,7} = H_m(W_7, v_1)$.

By the definitions of both corona product of graphs and the bridge graph $B_m = P_m \odot \overline{K}_2$, $T_{m,3} = P_m \odot K_2$ and $J_{m,n+1} = P_m \odot C_n$.

The graph $P_m \odot \overline{K}_n$ is a caterpillar (that is, a tree where the removal of all the end vertices results in a path.)

It can be easily verified that $DD(K_n) = n(n-1)^2$, $W(K_n) = \frac{1}{2}n(n-1)$, $M_1(S_n) = n(n-1)$ and $M_2(S_n) = (n-1)^2$. Using Theorem (2.3), Corollary (2.4), Lemmas (2.5)-(2.7), we obtain the exact degree distance of the graphs $P_m \odot \overline{K}_n$, $P_m \odot P_n$, $P_m \odot C_n$, $C_m \odot C_n$ and $P_m \odot S_n$.

(1) For $m \geq 2$, $n \geq 1$, $DD(P_m \odot \overline{K}_n) = \frac{1}{3}m(m-1)(n+1)(2mn+2m+2n-1) + mn(3mn+3m-4)$.

(2) For $m \geq 2$, $DD(P_m \odot \overline{K}_2) = 6m^3 + 15m^2 - 11m$.

(3) For $m \geq 2$, $n \geq 3$, $DD(P_m \odot P_n) = \frac{1}{3}\{4m^3n^2 + 4m^3n + 21m^2n^2 - 9m^2 - 4mn^2 - 43mn + 39m\}$.

(4) For $m \geq 2$, $DD(P_m \odot K_2) = 8m^3 + 25m^2 - 21m$.

(5) For $m \geq 2$, $n \geq 3$, $DD(P_m \odot C_n) = \frac{1}{3}\{4m^3n^2 + 6m^3n + 12m^2n + 21m^2n^2 + 2m^3 - 3m^2 - 4mn^2 - 45mn + m\}$.

(6) For $m \geq 2$, $DD(P_m \odot C_3) = \frac{1}{3}\{56m^3 + 222m^2 - 170m\}$.

(7) For $m \geq 3$, $n \geq 3$, $DD(C_m \odot C_n)$

$$= \begin{cases} \frac{1}{2}(n+1)(2n+1)m^3 + 7m^2n^2 + 5nm^2 - 12mn, & \text{if } m \text{ is even,} \\ \frac{1}{2}(n+1)(2n+1)m(m^2-1) + 7m^2n^2 + 5nm^2 - 12mn, & \text{if } m \text{ is odd.} \end{cases}$$

(8) For $m, n \geq 2$, $DD(P_m \odot S_n) = \frac{1}{3}\{4m^3n^2 + 4m^3n + 21m^2n^2 - 9m^2 - 7mn^2 - 28mn + 21m\}$.

3. Gutman Index of Corona Product of Graphs

In this section, we compute the Gutman index of the corona product $G_1 \odot G_2$ of the graphs G_1 and G_2.

Theorem 3.1. *Let G_1 and G_2 be arbitrary graphs. Then* $Gut(G_1 \odot G_2) = Gut(G_1) + 2\big[2\epsilon(G_2) + n_2\big]DD(G_1) + 4\big[\epsilon(G_2) + n_2\big]^2 W(G_1) + 4(n_1)^2\big(\epsilon(G_2)\big)^2 - n_1\big[2M_1(G_2) + M_2(G_2)\big] + 2n_1n_2\epsilon(G_1) + 4n_1\epsilon(G_1)\epsilon(G_2) + n_1\big(6n_1n_2 - 5\big)\epsilon(G_2) + 2(n_1n_2)^2 - n_1n_2$.

Proof. Let $G = G_1 \odot G_2$. Then

$$\begin{aligned}
Gut(G) &= \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)d_G(u)d_G(v)\\
&= \frac{1}{2}\Bigg\{\sum_{\substack{u_i,\,u_p\in V(G_1)\\ i\neq p}} d_G(u_i,u_p)d_G(u_i)d_G(u_p) + \sum_{i=0}^{n_1-1}\sum_{\substack{v_{ij},\,v_{iq}\in V(G_2^i)\\ j\neq q}} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq})\\
&\quad + 2\sum_{p=0}^{n_1-1}\sum_{\substack{u_i\in V(G_1),\\ v_{pq}\in V(G_2^p)}} d_G(u_i,v_{pq})d_G(u_i)d_G(v_{pq}) + \sum_{\substack{v_{ij}\in V(G_2^i),\\ v_{pq}\in V(G_2^p)\\ i\neq p}} d_G(v_{ij},v_{pq})\big[d_G(v_{ij}) + d_G(v_{pq})\big]\Bigg\}\\
&= \frac{1}{2}\Bigg\{\sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)d_G(u_i)d_G(u_p) + \sum_{i=0}^{n_1-1}\sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq})\\
&\quad + 2\sum_{p=0}^{n_1-1}\sum_{i=0}^{n_1-1}\sum_{q=0}^{n_2-1} d_G(u_i,v_{pq})d_G(u_i)d_G(v_{pq}) + \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1}\sum_{j,\,q=0}^{n_2-1} d_G(v_{ij},v_{pq})d_G(v_{ij})d_G(v_{pq})\Bigg\}\\
&= \frac{1}{2}\Big(A_1 + A_2 + A_3 + A_4\Big), \text{where } A_1\text{-}A_4 \text{ are the sums of the above terms, in order.} \qquad (3.1)
\end{aligned}$$

We calculate A_1-A_4 of ((3.1)) separately.

First we compute A_1.

$$\begin{aligned}
A_1 &= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)d_G(u_i)d_G(u_p)\\
&= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)\big[d_{G_1}(u_i) + n_2\big]\big[d_{G_1}(u_p) + n_2\big], \text{ by Lemmas (2.1) and (2.2),}\\
&= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)\Big\{d_{G_1}(u_i)d_{G_1}(u_p) + n_2\big[d_{G_1}(u_i) + d_{G_1}(u_p)\big] + (n_2)^2\Big\}\\
&= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)d_{G_1}(u_i)d_{G_1}(u_p) + n_2\sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)\big[d_{G_1}(u_i) + d_{G_1}(u_p)\big] + (n_2)^2\sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} d_G(u_i,u_p)\\
&= 2Gut(G_1) + 2n_2DD(G_1) + 2(n_2)^2W(G_1), \qquad (3.2)
\end{aligned}$$

by the definitions of Gutman index, degree distance and Wiener index of G_1, respectively.

Next we compute $A_2 = \sum_{i=0}^{n_1-1}\sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq})$. For this, initially we calculate $A_2' =$

$\sum\limits_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq}).$

$$\begin{aligned}
A_2' &= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq}) + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} d_G(v_{ij},v_{iq})d_G(v_{ij})d_G(v_{iq})\\
&= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big(d_{G_2}(v_j)+1\Big)\Big(d_{G_2}(v_q)+1\Big) + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} 2\Big(d_{G_2}(v_j)+1\Big)\Big(d_{G_2}(v_q)+1\Big)
\end{aligned}$$

by Lemmas (2.1) and (2.2),

$$\begin{aligned}
&= \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q) + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big] + 2\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q)\\
&\quad + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big]\\
&= \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q) + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big] + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q)\\
&\quad + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big]\Bigg\} + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q) + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big],
\end{aligned}$$

by splitting the second term into two terms.

$$\begin{aligned}
&= \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q) + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q)\Bigg\} + \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)\\
&\quad + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)\Bigg\} + \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} 1 + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} 1\Bigg\} + \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\notin E(G_2)}}^{n_2-1} \Big[d_{G_2}(v_j)d_{G_2}(v_q)\\
&\quad + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)+1\Big]\\
&= 4\big(\epsilon(G_2)\big)^2 - M_1(G_2) + 4(n_2-1)\epsilon(G_2) + 2\binom{n_2}{2} + \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q}}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q)\\
&\quad - \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q)\Bigg\} + \Bigg\{\sum_{\substack{j,\,q=0\\ j\neq q}} \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big) - \sum_{\substack{j,\,q=0\\ j\neq q\\ v_jv_q\in E(G_2)}}^{n_2-1} \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big)\Bigg\}\\
&\quad + 2\binom{n_2}{2} - 2\epsilon(G_2)\\
&= 2\Bigg\{4\big(\epsilon(G_2)\big)^2 - M_1(G_2) + 4(n_2-1)\epsilon(G_2) + 2\binom{n_2}{2}\Bigg\} - 2\Bigg\{\sum_{v_jv_q\in E(G_2)} d_{G_2}(v_j)d_{G_2}(v_q)\\
&\quad + \sum_{v_jv_q\in E(G_2)} \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big) + \epsilon(G_2)\Bigg\},
\end{aligned}$$

since both v_jv_q and v_qv_j are accounted in the second bracket terms.

$$= 8\big(\epsilon(G_2)\big)^2 - 2M_1(G_2) + 8(n_2-1)\epsilon(G_2) + 2n_2(n_2-1) - 2M_2(G_2) - 2M_1(G_2) - 2\epsilon(G_2)$$

$$= 8\big(\epsilon(G_2)\big)^2 - 4M_1(G_2) + 8n_2\epsilon(G_2) + 2n_2(n_2-1) - 2M_2(G_2) - 10\epsilon(G_2). \tag{3.3}$$

Now using ((3.3)), we get

$$A_2 = \sum_{i=0}^{n_1-1} \Big\{8\big(\epsilon(G_2)\big)^2 - 4M_1(G_2) + 8n_2\epsilon(G_2) + 2n_2(n_2-1) - 2M_2(G_2) - 10\epsilon(G_2)\Big\}$$

$$= n_1\Big\{8\big(\epsilon(G_2)\big)^2 - 4M_1(G_2) + 8n_2\epsilon(G_2) + 2n_2(n_2-1) - 2M_2(G_2) - 10\epsilon(G_2)\Big\}. \tag{3.4}$$

Next we compute $A_3 = 2\sum\limits_{p=0}^{n_1-1}\sum\limits_{i=0}^{n_1-1}\sum\limits_{q=0}^{n_2-1} d_G(u_i, v_{pq})d_G(u_i)d_G(v_{pq})$. For this first we compute $A_3' = \sum\limits_{i=0}^{n_1-1}\sum\limits_{q=0}^{n_2-1} d_G(u_i, v_{pq})d_G(u_i)d_G(v_{pq})$.

$$A_3' = \sum_{i=0}^{n_1-1}\sum_{q=0}^{n_2-1} \big[d_{G_1}(u_i,u_p)+1\big]\big[d_{G_1}(u_i)+n_2\big]\big[d_{G_2}(v_q)+1\big], \text{ by Lemmas (2.1) and (2.2).}$$

$$= \sum_{q=0}^{n_2-1}\big[d_{G_2}(v_q)+1\big]\Big\{\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)d_{G_1}(u_i) + n_2\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + \sum_{i=0}^{n_1-1} d_{G_1}(u_i)$$
$$+ \sum_{i=0}^{n_1-1} n_2\Big\}$$

$$= \big[2\epsilon(G_2)+n_2\big]\Big\{\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)d_{G_1}(u_i) + n_2\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + 2\epsilon(G_1) + n_1n_2\Big\}. \tag{3.5}$$

Using ((3.5)), we get

$$A_3 = 2\sum_{p=0}^{n_1-1}\bigg(\big[2\epsilon(G_2)+n_2\big]\Big\{\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p)d_{G_1}(u_i) + n_2\sum_{i=0}^{n_1-1} d_{G_1}(u_i,u_p) + 2\epsilon(G_1) + n_1n_2\Big\}\bigg)$$

$$= 2\big[2\epsilon(G_2)+n_2\big]\Big\{DD(G_1) + 2n_2W(G_1) + 2n_1\epsilon(G_1) + (n_1)^2n_2\Big\}, \tag{3.6}$$

from the definitions of degree distance and Wiener index of G_1, respectively.
Finally, we compute $A_4 = \sum\limits_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1}\sum\limits_{j,\,q=0}^{n_2-1} d_G(v_{ij}, v_{pq})d_G(v_{ij})d_G(v_{pq})$. For this, first we calculate $A_4' = \sum\limits_{j,\,q=0}^{n_2-1} d_G(v_{ij}, v_{pq})d_G(v_{ij})d_G(v_{pq})$.

$$A_4' = \sum_{j,\,q=0}^{n_2-1}\Big[d_{G_1}(u_i,u_p)+2\Big]\Big(d_{G_2}(v_j)+1\Big)\Big(d_{G_2}(v_q)+1\Big), \text{ by Lemmas (2.1) and (2.2).}$$

$$= \Big[d_{G_1}(u_i,u_p)+2\Big]\sum_{j,\,q=0}^{n_2-1}\Big[d_{G_2}(v_j)d_{G_2}(v_q) + \big(d_{G_2}(v_j)+d_{G_2}(v_q)\big) + 1\Big]$$

$$= \Big[d_{G_1}(u_i,u_p)+2\Big]\Big\{4\big(\epsilon(G_2)\big)^2 + 4n_2\epsilon(G_2) + (n_2)^2\Big\}, \tag{3.7}$$

since $\sum_{j,\,q=0}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q) = 4\big(\epsilon(G_2)\big)^2$ and $\sum_{j,\,q=0}^{n_2-1} d_{G_2}(v_j) + d_{G_2}(v_q) = 4n_2\epsilon(G_2)$.
Using ((3.7)), we get

$$\begin{aligned} A_4 &= \sum_{\substack{i,\,p=0\\ i\neq p}}^{n_1-1} \Big[d_{G_1}(u_i,u_p)+2\Big]\Big\{4\big(\epsilon(G_2)\big)^2 + 4n_2\epsilon(G_2) + (n_2)^2\Big\} \\ &= \Big\{4\big(\epsilon(G_2)\big)^2 + 4n_2\epsilon(G_2) + (n_2)^2\Big\}\Big(2W(G_1) + 2n_1(n_1-1)\Big), \end{aligned} \tag{3.8}$$

from the definition of Wiener index of G_1.
Using ((3.2)), ((3.4)), ((3.6)) and ((3.8)) in ((3.1)), we get

$$\begin{aligned} Gut(G) = \frac{1}{2}\Big\{ & 2Gut(G_1) + 2n_2DD(G_1) + 2(n_2)^2W(G_1) + n_1\Big[8\big(\epsilon(G_2)\big)^2 - 4M_1(G_2) \\ & + 8n_2\epsilon(G_2) + 2n_2(n_2-1) - 2M_2(G_2) - 10\epsilon(G_2)\Big] + 2\big[2\epsilon(G_2)+n_2\big]\Big[DD(G_1) \\ & + 2n_2W(G_1) + 2n_1\epsilon(G_1) + (n_1)^2n_2\Big] + \Big[4\big(\epsilon(G_2)\big)^2 + 4n_2\epsilon(G_2) \\ & + (n_2)^2\Big]\Big(2W(G_1) + 2n_1(n_1-1)\Big)\Big\} \\ = {} & Gut(G_1) + 2\big[2\epsilon(G_2)+n_2\big]DD(G_1) + 4\big[\epsilon(G_2)+n_2\big]^2W(G_1) + 4(n_1)^2\big(\epsilon(G_2)\big)^2 \\ & - n_1\big[2M_1(G_2) + M_2(G_2)\big] + 2n_1n_2\epsilon(G_1) + 4n_1\epsilon(G_1)\epsilon(G_2) + n_1\big(6n_1n_2 - 5\big)\epsilon(G_2) \\ & + 2(n_1n_2)^2 - n_1n_2. \end{aligned}$$

Corollary 3.2. *If G is a nontrivial connected graph with $|V(G)| = m$, then $Gut(G \odot \overline{K}_n) = Gut(G) + 2nDD(G) + 4n^2W(G) + 2mn\epsilon(G) + 2m^2n^2 - mn$.*

For our future reference we quote the following Lemmas.

Lemma 3.3. [7] *Let P_n and C_n denote the path and the cycle on n vertices, respectively. Then for $n \geq 2$, $Gut(P_n) = \frac{1}{3}(n-1)(2n^2-4n+3)$ and for $n \geq 3$,*

$$Gut(C_n) = \begin{cases} \dfrac{n^3}{2}, & \text{if } n \text{ is even} \\ \dfrac{n(n^2-1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

It can be easily verified that $Gut(K_n) = \frac{1}{2}n(n-1)^3$. Using Theorem (3.1), Corollary (3.2), Lemmas (2.5)-(2.7) and (3.3) we obtain the exact Gutman indices of the graphs $P_m \odot \overline{K}_n$, $P_m \odot P_n$, $P_m \odot C_n$, $C_m \odot C_n$ and $P_m \odot S_n$.

(1) For $m \geq 2$, $n \geq 1$, $Gut(P_m \odot \overline{K}_n) = \frac{1}{3}\big\{2m^3n^2 + 4m^3n + 2m^3 + 6m^2n^2 - 6m^2 - 2mn^2 - 7mn + 7m - 3\big\}$.

(2) For $m \geq 2$, $Gut(P_m \odot \overline{K}_2) = 6m^3 + 6m^2 - 5m - 1$.

(3) For $m \geq 2$, $n \geq 3$, $Gut(P_m \odot P_n) = \frac{1}{3}\big\{8m^3n^2 + 36m^2n^2 - 36m^2n - 8mn^2 - 60mn + 90m - 3\big\}$.

(4) For $m \geq 2$, $Gut(P_m \odot K_2) = \frac{1}{3}\big\{32m^3 + 72m^2 - 65m - 3\big\}$.

(5) For $m \geq 2$, $n \geq 3$, $Gut(P_m \odot C_n) = \frac{1}{3}\big\{8m^3n^2 + 8m^3n + 6m^2n + 36m^2n^2 + 2m^3 - 6m^2 - 8mn^2 - 68mn + 7m - 3\big\}$.

(6) For $m \geq 2$, $Gut(P_m \odot C_3) = \frac{1}{3}\big\{98m^3 + 336m^2 - 269m - 3\big\}$.

(7) For $m \geq 3$, $n \geq 3$, $Gut(C_m \odot C_n)$
$$= \begin{cases} \frac{1}{2}m^3(2n+1)^2 + 12m^2n^2 + 6m^2n - 18mn, & \text{if } m \text{ is even,} \\ \frac{1}{2}m(m^2-1)(2n+1)^2 + 12m^2n^2 + 6m^2n - 18mn, & \text{if } m \text{ is odd.} \end{cases}$$

(8) For $m, n \geq 2$, $Gut(P_m \odot S_n) = \frac{1}{3}\{8m^3n^2 - 36m^2n + 36m^2n^2 - 17mn^2 - 12mn + 27m\}$.

4. Conclusion

In this paper, we have studied the degree distance and Gutman index of the corona product, $G \odot H$. Using the results obtained for $G \odot H$, the exact degree distance and Gutman index of the caterpillar are obtained; based on the formulae for the degree distance and Gutman index obtained for $G \odot H$, the exact degree distance and Gutman indices of the graphs $P_m \odot \overline{K}_n$, $P_m \odot P_n$, $P_m \odot C_n$, $C_m \odot C_n$ and $P_m \odot S_n$ are obtained.

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