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A NOTE ON STAR COLORING OF CENTRAL GRAPH OF BIPARTITE GRAPH AND CORONA GRAPH OF COMPLETE GRAPH WITH PATH AND CYCLE

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Abstract. In this paper, we find the star chromatic number of central graph of complete bipartite graph and corona graph of complete graph with path and cycle.

1. Introduction

The notion of star chromatic number was introduced by Branko Grünbaum in 1973.

A star coloring [1, 2, 3] of a graph G is a proper vertex coloring in which every path on four vertices uses at least three distinct colors. Equivalently, in a star coloring, the induced subgraphs formed by the vertices of any two colors has connected components that are star graphs. The star chromatic number $\chi_s(G)$ of G is the least number of colors needed to star color G.

A number of results exist for star colorings of graphs formed by certain graph operations. Guillaume Fertin et al.[3] gave the exact value of the star chromatic number of different families of graphs such as trees, cycles, complete bipartite graphs, outerplanar graphs, and 2-dimensional grids. They also investigated and gave bounds for the star chromatic number of other families of graphs, such as planar graphs, hypercubes, d-dimensional grids ($d \geq 3$), d-dimensional tori ($d \geq 2$), graphs with bounded treewidth, and cubic graphs.

Albertson et al. [1] showed that it is NP-complete to determine whether $\chi_s(G) \leq 3$, even when G is a graph that is both planar and bipartite. The problems of finding star colorings is NP-hard and remain so even for bipartite graphs [4, 5] .

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In Section 3, we find the star chromatic number of central graph of complete bipartite graph.

In Section 4, we find the star chromatic number of corona graph of complete graph with path and cycle.

2. Preliminaries

All graphs considered are loopless graphs without multiple edges. A path on n vertices will be denoted by P_n. A cycle on n vertices will be denoted by C_n.

For a given graph $G = (V, E)$ we do an operation on G, by subdividing each edge exactly once and joining all the non adjacent vertices of G. The graph obtained by this process is called central graph [6, 7] of G denoted by $C(G)$.

The corona [8] of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2.

3. Star coloring on central graph of complete bipartite graph

Theorem 3.1. *Let $K_{m,n}$ be a complete bipartite graph on m and n vertices. Then*

$$\chi_s(C(K_{m,n})) = \begin{cases} m; & \text{if } m \geq n, \\ n; & \text{otherwise.} \end{cases}$$

Proof. Let $\{v_i : 1 \leq i \leq n\}$ and $\{u_j : 1 \leq j \leq m\}$ be the vertices of $K_{m,n}$ and by the definition of complete bipartite graph, every vertex from $\{v_i : 1 \leq i \leq n\}$ is adjacent to every vertex from the set $\{u_j : 1 \leq j \leq m\}$. Let $\{e_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ be the set of edges of $K_{m,n}$.

By the definition of central graph, the edges $\{e_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ be subdivided by the vertex $\{w_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ in $C(K_{m,n})$, and let $V = \{v_1, v_2, \ldots, v_n\}$, $V' = \{u_1, u_2, \ldots, u_m\}$. Clearly $V(C(K_{m,n}))= V \cup V' \cup \{w_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$. Note that in $C(K_{m,n})$, the induced subgraphs $\langle\{v_1, v_2, \ldots, v_n\}\rangle$ and $\langle\{u_1, u_2, \ldots, u_m\}\rangle$ are complete. Therefore,

$$\chi_s(C(K_{m,n})) \geq \begin{cases} m; & \text{if } m \geq n, \\ n; & \text{if } n \geq m. \end{cases}$$

The following coloring for $C(K_{m,n})$ is star chromatic: For $1 \leq i \leq n$, assign the color c_i for v_i. For $1 \leq j \leq m$, assign the color c_j for u_j. For $1 \leq i \leq n$ and $1 \leq j \leq m$, assign to vertex $\{w_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ one of allowed colors - such color exists, because $deg(w_{ij}) = 2$. Thus we have,

$$\chi_s(C(K_{m,n})) \leq \begin{cases} m; & \text{if } m \geq n, \\ n; & \text{if } n \geq m. \end{cases}$$

Hence,

$$\chi_s(C(K_{m,n})) = \begin{cases} m; & \text{if } m \geq n, \\ n; & otherwise. \end{cases}$$

$\square$

4. Star coloring on corona graph of complete graph with path and cycle

Theorem 4.1. *Let K_n be a complete graph on n vertices. Then*

$$\chi_s(K_n \circ P_n) = n, \ \forall \ n \geq 3.$$

Proof. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and $V(P_n) = \{u_1, u_2, \ldots, u_n\}$. Let $V(K_n \circ P_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$. By the definition of corona graph, each vertex of G is adjacent to every vertex of a copy of P_n. i.e., every vertex $v_i \in V(K_n)$ is adjacent to every vertex from the set $\{u_{ij} : \ 1 \leq j \leq n\}$. Note that $\chi_s(K_n \circ P_n) \geq n$, since K_n is n chromatic.

Assign the following star coloring for $K_n \circ P_n$ as star-chromatic:

- For $1 \leq i \leq n$, assign the color c_i to v_i.
- For $1 \leq i \leq n$, assign the color c_i to $u_{1i}, \ \forall \ i \neq 1$.
- For $1 \leq i \leq n$, assign the color c_i to $u_{2i}, \ \forall \ i \neq 2$.
- For $1 \leq i \leq n$, assign the color c_i to $u_{3i}, \ \forall \ i \neq 3$.
- For $1 \leq i \leq n$, assign the color c_i to $u_{4i}, \ \forall \ i \neq 4$.

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- For $1 \leq i \leq n$, assign the color c_i to $u_{ni}, \forall \ i \neq n$.
- For $1 \leq i \leq n$, assign to vertex u_{ii} one of allowed colors - such color exists, because $2 \leq deg(u_{ii}) \leq 3$ and $n \geq 3$.

Thus we have, $\chi_s(K_n \circ P_n) \leq n$. Hence, $\chi_s(K_n \circ P_n) = n, \ \forall \ n \geq 3.$ □

Theorem 4.2. *Let K_n be a complete graph on n vertices. Then*

$$\chi_s(K_n \circ C_n) = n, \ \forall \ n \geq 3.$$

Proof. The proof is similar to the proof of Theorem 4.1 □

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References

[1] Michael O. Albertson, Glenn G. Chappell, H. A. Kierstead, André Kündgen and Radhika Ramamurthi, *Coloring with no 2-colored P_4's*, The Electronic Journal of Combinatorics, **11** (2004), Paper # R26, 13.

[2] Branko Grünbaum, *Acyclic colorings of planar graphs*, Israel J. Math., **14** (1973), 390–408.

[3] Guillaume Fertin, André Raspaud and Bruce Reed, *Star coloring of graphs*, J. Graph Theory, **47** (3) (2004), 163–182.

[4] Thomas F. Coleman and Jin-Yi Cai, *The cyclic coloring problem and estimation of sparse Hessian matrices*, SIAM J. Alg. Disc. Math. 7 (1986), 221–235.

[5] Thomas F. Coleman and Jorge J. Moré, *Estimation of sparse Hessian matrices and graph coloring problems*, Mathematical Programming, **28** (3) (1984), 243–270.

[6] Vivin J. Vernold, *Harmonious coloring of total graphs, n-leaf, central graphs and circumdetic graphs*, Ph. D. Thesis, Bharathiar University, (2007), Coimbatore, India.

[7] Vivin J. Vernold, M. Venkatachalam and Ali M. M. Akbar, *A note on achromatic coloring of star graph families*, Filomat, **23** (3) (2009), 251–255.

[8] Vivin J. Vernold and M. Venkatachalam, *The b-chromatic number of corona graphs*, Utilitas Mathematica (to appear).

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