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MODULAR EDGE COLORINGS OF MYCIELSKIAN GRAPHS

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Abstract. Let G be a connected graph of order 3 or more and $c : E(G) \to \mathbb{Z}_k$ $(k \geq 2)$ a k-edge coloring of G where adjacent edges may be colored the same. The color sum $s(v)$ of a vertex v of G is the sum in $\mathbb{Z}_k$ of the colors of the edges incident with v. The k-edge coloring c is a modular k-edge coloring of G if $s(u) \neq s(v)$ in $\mathbb{Z}_k$ for all pairs u, v of adjacent vertices of G. The modular chromatic index $\chi'_m(G)$ of G is the minimum k for which G has a modular k-edge coloring. The Mycielskian of $G = (V, E)$ is the graph $\mathscr{M}(G)$ with vertex set $V \cup V' \cup \{u\}$, where $V' = \{v' : v \in V\}$, and edge set $E \cup \{xy' : xy \in E\} \cup \{v'u : v' \in V'\}$. It is shown that $\chi'_m(\mathscr{M}(G)) = \chi(\mathscr{M}(G))$ for some bipartite graphs, cycles and complete graphs.

1. Introduction

For a connected graph G of order 3 or more, let $c : E(G) \to \mathbb{Z}_k$ $(k \geq 2)$ be a k-edge coloring of G where adjacent edges may be colored the same. The *color sum* $s(v)$ of a vertex v of G is defined as the sum in $\mathbb{Z}_k$ of the colors of the edges incident with v, that is, if E_v is the set of edges incident with v in G, then $s(v) = \sum_{e \in E_v} c(e)$. The k-edge coloring c is a *modular k-edge coloring* of G if $s(u) \neq s(v)$ in $\mathbb{Z}_k$ for all pairs u, v of adjacent vertices of G. An edge coloring c is a modular edge coloring if c is a modular k-edge coloring for some integer $k \geq 2$. The *modular chromatic index* $\chi'_m(G)$ of G is the minimum k for which G has a modular k-edge coloring. If G contains a component isomorphic to K_2, say $V(K_2) = \{u, v\}$, then $s(u) = s(v)$ for any edge coloring c of G, which implies that G does not have a modular edge coloring. On the other hand, every graph containing neither isolated vertices nor

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components isomorphic to K_2 has a modular edge coloring. This concept was introduced by Jones, Kolasinski, Okamoto and Zhang [2].

Jones, Kolasinski, Okamoto and Zhang proved in [2] that: for every connected graph G of order at least 3, $\chi'_m(G) \geq \chi(G)$; for each integer $n \geq 3$, $\chi'_m(K_n)$ is $n+1$ if $n \equiv 2 \mod 4$ and it is n otherwise; for each integer $n \geq 3$, $\chi'_m(K_n - e) = n - 1$; for each integer $n \geq 3$, $\chi'_m(C_n)$ is 2 if $n \equiv 0 \mod 4$ and it is 3 otherwise; let G be a graph such that $\chi(G) \equiv 2 \mod 4$, if each color class in every proper $\chi(G)$-coloring of G consists of an odd number of vertices, then $\chi'_m(G) > \chi(G)$; for each integer $n \geq 3$, $\chi'_m(P_n)$ is 3 if $n \equiv 2 \mod 4$ and it is 2 otherwise; for positive integers r and s where $r + s \geq 3$, $\chi'_m(K_{r,s})$ is 3 if r and s are odd and it is 2 otherwise; let T be a tree of order $r + s \geq 3$ whose partite sets have orders r and s, then $\chi'_m(T)$ is 3 if r and s are odd and it is 2 otherwise; if G is a connected bipartite graph of order $r + s \geq 3$ such that $G \subseteq K_{r,s}$, then $\chi'_m(G)$ is 3 if r and s are odd and it is 2 otherwise.

The *Mycielskian* of $G = (V, E)$ is the graph $\mathscr{M}(G)$ with vertex set $V \cup V' \cup \{u\}$, where $V' = \{v' : v \in V\}$, and edge set $E \cup \{xy' : xy \in E\} \cup \{v'u : v' \in V'\}$.

In this paper we determine modular chromatic indices of several Mycielskians.

2. **Results**

We begin with bipartite graphs.

Theorem 2.1. *Let G be a bipartite graph of order 3 or more. If G contains a Hamilton path, then*

$$\chi'_m(\mathscr{M}(G)) = \chi(\mathscr{M}(G)) = 3.$$

Proof. Let $n = |V(G)|$, (X, Y) be the bipartition of G, and P be a Hamilton path of G. Without loss of generality assume that one end of P is in X. We consider two cases:

Case 1. $n \equiv 0 \mod 2$.

Then $P = x_1y_1x_2y_2 \ldots x_{\frac{n}{2}}y_{\frac{n}{2}}$, where $X = \{x_1, x_2, \ldots, x_{\frac{n}{2}}\}$ and $Y = \{y_1, y_2, \ldots, y_{\frac{n}{2}}\}$. Define $c : E(\mathscr{M}(G)) \to \mathbb{Z}_3$ as follows:

For $n \equiv 0 \mod 4$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i-1}y_{2i-1}, y_{2i-1}x_{2i} : i \in \{1, 2, \ldots, \frac{n}{4}\}\} \bigcup \{x'_ju : j \in \{1, 2, \ldots, \frac{n}{2}\}\}, \\ 2 & \text{if } e \in \{y'_ju : j \in \{1, 2, \ldots, \frac{n}{2}\}\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1, 2, \ldots, \frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{y_{2i-1} : i \in \{1, 2, \ldots, \frac{n}{4}\}\} \bigcup \{y'_j : j \in \{1, 2, \ldots, \frac{n}{2}\}\}, \\ 0 & \text{if } v \in \{y_{2i} : i \in \{1, 2, \ldots, \frac{n}{4}\}\} \text{ or } v = u. \end{cases}$$

For $n \equiv 2 \mod 12$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\} \bigcup \{x'_j u : j \in \{1,2,\ldots,\frac{n}{2}\}\} \bigcup \{y'_1 u\}, \\ 2 & \text{if } e = x_1 y'_1, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j : j \in \{2,3,\ldots,\frac{n}{2}\}\} \bigcup \{x'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{x_1, u\} \bigcup \{y_{2i} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\}, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{1,2,\ldots,\frac{n+2}{4}\}\} \bigcup \{y'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}. \end{cases}$$

For $n \equiv 6 \mod 12$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\} \bigcup \{x'_j u : j \in \{1,2,\ldots,\frac{n}{2}\}\} \bigcup \{x_1 y'_1\}, \\ 2 & \text{if } e = y'_1 u, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{y_{2i} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\} \text{ or } v = u, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{1,2,\ldots,\frac{n+2}{4}\}\} \bigcup \{y'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}. \end{cases}$$

For $n \equiv 10 \mod 12$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\} \bigcup \{x'_j u : j \in \{2,3,\ldots,\frac{n}{2}\}\} \bigcup \{x_1 y'_1\}, \\ 2 & \text{if } e \in \{y_1 x'_1, x'_1 u, y'_1 u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{u, y_1\} \bigcup \{y_{2i} : i \in \{1,2,\ldots,\frac{n-2}{4}\}\}, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{2,3,\ldots,\frac{n+2}{4}\}\} \bigcup \{y'_j : j \in \{1,2,\ldots,\frac{n}{2}\}\}. \end{cases}$$

Case 2. $n \equiv 1 \mod 2$.

Then $P = x_1, y_1, x_2, y_2 \ldots, x_{\frac{n-1}{2}} y_{\frac{n-1}{2}} x_{\frac{n+1}{2}}$, where $X = \{x_1, x_2, \ldots, x_{\frac{n+1}{2}}\}$ and $Y = \{y_1, y_2, \ldots, y_{\frac{n-1}{2}}\}$. Define $c : E(\mathscr{M}(G)) \to \mathbb{Z}_3$ as follows:

For $n \equiv 1 \mod 4$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{y_{2i-1}x_{2i}, x_{2i}y_{2i} : i \in \{1,2,\ldots,\frac{n-1}{4}\}\} \bigcup \{y'_j u : j \in \{1,2,\ldots,\frac{n-3}{2}\}\}, \\ 2 & \text{if } e \in \{x'_j u : j \in \{1,2,\ldots,\frac{n+1}{2}\}\} \bigcup \{x_{\frac{n+1}{2}} y'_{\frac{n-1}{2}}, y'_{\frac{n-1}{2}} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \ldots, \frac{n-1}{2}\}\}, \\ 2 & \text{if } v \in \{x_{2i} : i \in \{1, 2, \ldots, \frac{n-1}{4}\} \bigcup \{x_{\frac{n+1}{2}}\} \bigcup \{x'_j : j \in \{1, 2, \ldots, \frac{n+1}{2}\}\}, \\ 0 & \text{if } v \in \{x_{2i-1} : i \in \{1, 2, \ldots, \frac{n-1}{4}\}\} \text{ or } v = u. \end{cases}$$

For $n \equiv 3 \mod 4$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{y_{2i-1}x_{2i}, x_{2i}y_{2i} : i \in \{1, 2, \ldots, \frac{n-3}{4}\}\} \bigcup \{y_{\frac{n-1}{2}} x_{\frac{n+1}{2}}\} \bigcup \\ & \qquad \{y'_j u : j \in \{1, 2, \ldots, \frac{n-3}{2}\}\}, \\ 2 & \text{if } e \in \{x'_j u : j \in \{1, 2, \ldots, \frac{n+1}{2}\}\} \bigcup \{x_{\frac{n+1}{2}} y'_{\frac{n-1}{2}}, y'_{\frac{n-1}{2}} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \ldots, \frac{n-1}{2}\}\}, \\ 2 & \text{if } v \in \{x_{2i} : i \in \{1, 2, \ldots, \frac{n-3}{4}\} \bigcup \{x'_j : j \in \{1, 2, \ldots, \frac{n+1}{2}\}\}, \\ 0 & \text{if } v \in \{x_{2i-1} : i \in \{1, 2, \ldots, \frac{n+1}{4}\}\} \bigcup \{x_{\frac{n+1}{2}}\} \text{ or } v = u. \end{cases}$$

This completes the proof. □

Corollary 2.2. *For each integer $n \geq 3$,*

$$\chi'_m(\mathscr{M}(P_n)) = \chi(\mathscr{M}(P_n)) = 3.$$

Corollary 2.3. *For each integer $n \geq 2$,*

$$\chi'_m(\mathscr{M}(C_{2n})) = \chi(\mathscr{M}(C_{2n})) = 3.$$

Theorem 2.4. *Let G be a bipartite graph of order 3 or more. If G contains a vertex that is adjacent to all the vertices in a partite set of G, then*

$$\chi'_m(\mathscr{M}(G)) = \chi(\mathscr{M}(G)) = 3.$$

Proof. Let (X, Y) be the bipartition of G and let $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. By hypothesis, G contains a vertex, say, x_1 such that x_1 is adjacent to all the vertices in Y. Define $c : E(\mathscr{M}(G)) \to \mathbb{Z}_3$ as follows:

For $n \equiv 0 \mod 3$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j : j \in \{4, 5, \ldots, n\}\} \bigcup \{y'_j u : j \in \{1, 2, \ldots, n\}\} \\ & \qquad \bigcup \{y_j x'_1 : j \in \{2, 3, \ldots, n\}\}, \\ 2 & \text{if } e \in \{y_1 x'_1, x'_1 u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_2, y_3\} \bigcup \{y'_j : j \in \{1, 2, \ldots, n\}\}, \\ 2 & \text{if } v \in \{y_1\} \bigcup \{y_j : j \in \{4, 5, \ldots, n\}\} \bigcup \{u\}, \\ 0 & \text{if } v \in \{x_i, x'_i : i \in \{1, 2, \ldots, m\}\}. \end{cases}$$

(For $n = 3$, $\{4, 5, \ldots, n\} = \emptyset$.)

For $n \equiv 1 \mod 3$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j, y_j x'_1 : j \in \{2, 3, \ldots, n\}\} \bigcup \{y'_j u : j \in \{1, 2, \ldots, n\}\} \bigcup \{x'_1 u\}, \\ 2 & \text{if } e = y_1 x'_1, \\ 0 & \text{otherwise}, \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y'_j : j \in \{1, 2, \ldots, n\}\}, \\ 2 & \text{if } v \in \{y_j : j \in \{1, 2, \ldots, n\}\} \bigcup \{u\}, \\ 0 & \text{if } v \in \{x_i, x'_i : i \in \{1, 2, \ldots, m\}\}. \end{cases}$$

For $n \equiv 2 \mod 3$,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j, y'_j u : j \in \{1, 2, \ldots, n\}\}, \\ 0 & \text{otherwise}, \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \ldots, n\}\}, \\ 2 & \text{if } v \in \{x_1, u\}, \\ 0 & \text{if } v \in \{x_i : i \in \{2, 3, \ldots, m\}\} \bigcup \{x'_i : i \in \{1, 2, \ldots, m\}\}. \end{cases}$$

This completes the proof. □

Corollary 2.5. *For positive integers r and s with $r + s \geq 3$,*

$$\chi'_m(\mathscr{M}(K_{r,s})) = \chi(\mathscr{M}(K_{r,s})) = 3.$$

Next, we consider odd cycles.

Theorem 2.6. *For each integer $n \geq 2$,*

$$\chi'_m(\mathscr{M}(C_{2n+1})) = \chi(\mathscr{M}(C_{2n+1})) = 4.$$

Proof. Let $C_{2n+1} = v_1 v_2 v_3 \ldots v_{2n+1} v_1$. Define $c : E(\mathscr{M}(C_{2n+1})) \to \mathbb{Z}_4$ as follows: We consider four cases:

Case 1. $n \equiv 0 \mod 4$.

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3,7,11,\ldots,2n-1\} \cup \{4,8,12,\ldots,2n\}\} \bigcup \\ & \qquad \{v'_i u : i \in \{1,3,5,\ldots,2n+1\}\}, \\ 2 & \text{if } e \in \{v_{2n+1}v_1, v'_{2n}u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{3,5,7,\ldots,2n-1\}\} \bigcup \{v'_i : i \in \{1,3,5,\ldots,2n+1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4,8,12,\ldots,2n\}\} \bigcup \{v_1, v'_{2n}\}, \\ 3 & \text{if } v \in \{v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2,6,10,\ldots,2n-2\}\} \bigcup \{v'_i : i \in \{2,4,6,\ldots,2n-2\}\}. \end{cases}$$

Case 2. $n \equiv 1 \mod 4$ and $n \geq 5$.

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3,7,11,\ldots,2n-3\} \cup \{4,8,12,\ldots,2n-2\}\} \bigcup \\ & \qquad \{v'_i u : i \in \{1,3,5,\ldots,2n-1\}\} \bigcup \{v_{2n+1}v_1\}, \\ 2 & \text{if } e \in \{v_{2n-1}v_{2n}, v_{2n}v_{2n+1}, v'_{2n+1}u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{1,3,5,\ldots,2n-3\}\} \bigcup \{v'_i : i \in \{1,3,5,\ldots,2n-1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4,8,12,\ldots,2n-2\}\} \cup \{v'_{2n+1}\}, \\ 3 & \text{if } v \in \{v_{2n-1}, v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2,6,10,\ldots,2n\}\} \bigcup \{v'_i : i \in \{2,4,6,\ldots,2n\}\}. \end{cases}$$

Case 3. $n \equiv 2 \mod 4$.

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3,7,11,\ldots,2n-1\} \cup \{4,8,12,\ldots,2n\}\} \bigcup \\ & \qquad \{v'_i u : i \in \{1,3,5,\ldots,2n+1\}\}, \\ 2 & \text{if } e \in \{v_{2n+1}v_1\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{3,5,7,\ldots,2n-1\}\} \bigcup \{v'_i : i \in \{1,3,5,\ldots,2n+1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4,8,12,\ldots,2n\}\} \bigcup \{v_1\}, \\ 3 & \text{if } v \in \{v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2,6,10,\ldots,2n-2\}\} \bigcup \{v'_i : i \in \{2,4,6,\ldots,2n\}\}. \end{cases}$$

Case 4. $n \equiv 3 \mod 4$.

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_iv_{i+1} : i \in \{3,7,11,\ldots,2n-3\} \cup \{4,8,12,\ldots,2n-2\}\} \bigcup \\ & \qquad \{v'_iu : i \in \{1,3,5,\ldots,2n-1\}\} \bigcup \{v_{2n+1}v_1\}, \\ 2 & \text{if } e \in \{v_{2n-1}v_{2n}, v_{2n}v_{2n+1}, v'_{2n}u, v'_{2n+1}u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{1,3,5,\ldots,2n-3\}\} \bigcup \{v'_i : i \in \{1,3,5,\ldots,2n-1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4,8,12,\ldots,2n-2\}\} \bigcup \{v'_{2n}, v'_{2n+1}\}, \\ 3 & \text{if } v \in \{v_{2n-1}, v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2,6,10,\ldots,2n\}\} \bigcup \{v'_i : i \in \{2,4,6,\ldots,2n-2\}\}. \end{cases}$$

This completes the proof. □

Finally, we consider complete graphs.

Theorem 2.7. *For each integer $n \geq 3$,*

$$\chi'_m(\mathscr{M}(K_n)) = \chi(\mathscr{M}(K_n)) = n+1.$$

Proof. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. Define $c : E(\mathscr{M}(K_n)) \to \mathbb{Z}_{n+1}$ as follows:

For $n = 2r+1$,

$$c(e) = \begin{cases} i & \text{if } e = v_iv_{2r+1} \text{ for } i \in \{1,2,\ldots,2r\}, \\ i & \text{if } e = v'_iu \text{ for } i \in \{2,3,\ldots,2r-1\}, \\ 2r+1 & \text{if } e = v'_{2r+1}u, \\ r+1 & \text{if } e = v'_{2r}u, \\ r-1 & \text{if } e = v_{2r+1}v'_{2r}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i & \text{if } v = v_i \text{ for } i \in \{1,2,\ldots,2r+1\}, \\ i & \text{if } v = v'_i \text{ for } i \in \{2,3,\ldots,2r+1\}, \\ 1 & \text{if } v = u, \\ 0 & \text{if } v = v'_1. \end{cases}$$

For $n = 4r$,

$$c(e) = \begin{cases} 4i-3 & \text{if } e = v_{4i-3}v_{4i-2} \text{ for } i \in \{1,2,\ldots,r\}, \\ 1 & \text{if } e = v_{4i-2}v_{4i} \text{ for } i \in \{1,2,\ldots,r\}, \\ 4i-1 & \text{if } e = v_{4i-1}v_{4i} \text{ for } i \in \{1,2,\ldots,r\}, \\ i & \text{if } e = v_i'u \text{ for } i \in \{1,2,\ldots,4r\}, \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i & \text{if } v = v_i \text{ for } i \in \{1,2,\ldots,4r\}, \\ i & \text{if } v = v_i' \text{ for } i \in \{1,2,\ldots,4r\}, \\ 0 & \text{if } v = u. \end{cases}$$

For $n = 4r+2$,

$$c(e) = \begin{cases} i-1 & \text{if } e = v_iv_{4r+2} \text{ for } i \in \{2,3,\ldots,2r+1\}, \\ i+1 & \text{if } e = v_iv_{4r+2} \text{ for } i \in \{2r+2,2r+3,\ldots,4r+1\}, \\ i+1 & \text{if } e = v_i'u \text{ for } i \in \{1,2,\ldots,4r+1\}, \\ 1 & \text{if } e \in \{v_iv_{i+1} : i \in \{1,2,\ldots,2r\}\} \bigcup \{v_1v_{2r+1}, v_{4r+2}'u\}, \\ 0 & \text{otherwise}, \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i+1 & \text{if } v = v_i \text{ for } i \in \{1,2,\ldots,4r+1\}, \\ i+1 & \text{if } v = v_i' \text{ for } i \in \{1,2,\ldots,4r+1\}, \\ 1 & \text{if } v = v_{4r+2}', \\ 0 & \text{if } v \in \{v_{4r+2}, u\}. \end{cases}$$

This completes the proof. □

3. Conclusion

For every graph G considered in this paper, we have seen that $\chi'_m(\mathscr{M}(G)) = \chi(\mathscr{M}(G))$.

Problem 3.1. *Is it true that*

$$\chi'_m(\mathscr{M}(G)) = \chi(\mathscr{M}(G))$$

for every connected graph G of order 3 or more?

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