

## MODULAR EDGE COLORINGS OF MYCIELSKIAN GRAPHS

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ABSTRACT. Let  $G$  be a connected graph of order 3 or more and  $c : E(G) \rightarrow \mathbb{Z}_k$  ( $k \geq 2$ ) a  $k$ -edge coloring of  $G$  where adjacent edges may be colored the same. The color sum  $s(v)$  of a vertex  $v$  of  $G$  is the sum in  $\mathbb{Z}_k$  of the colors of the edges incident with  $v$ . The  $k$ -edge coloring  $c$  is a modular  $k$ -edge coloring of  $G$  if  $s(u) \neq s(v)$  in  $\mathbb{Z}_k$  for all pairs  $u, v$  of adjacent vertices of  $G$ . The modular chromatic index  $\chi'_m(G)$  of  $G$  is the minimum  $k$  for which  $G$  has a modular  $k$ -edge coloring. The Mycielskian of  $G = (V, E)$  is the graph  $\mathcal{M}(G)$  with vertex set  $V \cup V' \cup \{u\}$ , where  $V' = \{v' : v \in V\}$ , and edge set  $E \cup \{xy' : xy \in E\} \cup \{v'u : v' \in V'\}$ . It is shown that  $\chi'_m(\mathcal{M}(G)) = \chi(\mathcal{M}(G))$  for some bipartite graphs, cycles and complete graphs.

### 1. Introduction

For a connected graph  $G$  of order 3 or more, let  $c : E(G) \rightarrow \mathbb{Z}_k$  ( $k \geq 2$ ) be a  $k$ -edge coloring of  $G$  where adjacent edges may be colored the same. The *color sum*  $s(v)$  of a vertex  $v$  of  $G$  is defined as the sum in  $\mathbb{Z}_k$  of the colors of the edges incident with  $v$ , that is, if  $E_v$  is the set of edges incident with  $v$  in  $G$ , then  $s(v) = \sum_{e \in E_v} c(e)$ . The  $k$ -edge coloring  $c$  is a *modular  $k$ -edge coloring* of  $G$  if  $s(u) \neq s(v)$  in  $\mathbb{Z}_k$  for all pairs  $u, v$  of adjacent vertices of  $G$ . An edge coloring  $c$  is a modular edge coloring if  $c$  is a modular  $k$ -edge coloring for some integer  $k \geq 2$ . The *modular chromatic index*  $\chi'_m(G)$  of  $G$  is the minimum  $k$  for which  $G$  has a modular  $k$ -edge coloring. If  $G$  contains a component isomorphic to  $K_2$ , say  $V(K_2) = \{u, v\}$ , then  $s(u) = s(v)$  for any edge coloring  $c$  of  $G$ , which implies that  $G$  does not have a modular edge coloring. On the other hand, every graph containing neither isolated vertices nor

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components isomorphic to  $K_2$  has a modular edge coloring. This concept was introduced by Jones, Kolasinski, Okamoto and Zhang [2].

Jones, Kolasinski, Okamoto and Zhang proved in [2] that: for every connected graph  $G$  of order at least 3,  $\chi'_m(G) \geq \chi(G)$ ; for each integer  $n \geq 3$ ,  $\chi'_m(K_n)$  is  $n + 1$  if  $n \equiv 2 \pmod 4$  and it is  $n$  otherwise; for each integer  $n \geq 3$ ,  $\chi'_m(K_n - e) = n - 1$ ; for each integer  $n \geq 3$ ,  $\chi'_m(C_n)$  is 2 if  $n \equiv 0 \pmod 4$  and it is 3 otherwise; let  $G$  be a graph such that  $\chi(G) \equiv 2 \pmod 4$ , if each color class in every proper  $\chi(G)$ -coloring of  $G$  consists of an odd number of vertices, then  $\chi'_m(G) > \chi(G)$ ; for each integer  $n \geq 3$ ,  $\chi'_m(P_n)$  is 3 if  $n \equiv 2 \pmod 4$  and it is 2 otherwise; for positive integers  $r$  and  $s$  where  $r + s \geq 3$ ,  $\chi'_m(K_{r,s})$  is 3 if  $r$  and  $s$  are odd and it is 2 otherwise; let  $T$  be a tree of order  $r + s \geq 3$  whose partite sets have orders  $r$  and  $s$ , then  $\chi'_m(T)$  is 3 if  $r$  and  $s$  are odd and it is 2 otherwise; if  $G$  is a connected bipartite graph of order  $r + s \geq 3$  such that  $G \subseteq K_{r,s}$ , then  $\chi'_m(G)$  is 3 if  $r$  and  $s$  are odd and it is 2 otherwise.

The *Mycielskian* of  $G = (V, E)$  is the graph  $\mathcal{M}(G)$  with vertex set  $V \cup V' \cup \{u\}$ , where  $V' = \{v' : v \in V\}$ , and edge set  $E \cup \{xy' : xy \in E\} \cup \{v'u : v' \in V'\}$ .

In this paper we determine modular chromatic indices of several Mycielskians.

## 2. Results

We begin with bipartite graphs.

**Theorem 2.1.** *Let  $G$  be a bipartite graph of order 3 or more. If  $G$  contains a Hamilton path, then*

$$\chi'_m(\mathcal{M}(G)) = \chi(\mathcal{M}(G)) = 3.$$

*Proof.* Let  $n = |V(G)|$ ,  $(X, Y)$  be the bipartition of  $G$ , and  $P$  be a Hamilton path of  $G$ . Without loss of generality assume that one end of  $P$  is in  $X$ . We consider two cases:

*Case 1.*  $n \equiv 0 \pmod 2$ .

Then  $P = x_1y_1x_2y_2 \dots x_{\frac{n}{2}}y_{\frac{n}{2}}$ , where  $X = \{x_1, x_2, \dots, x_{\frac{n}{2}}\}$  and  $Y = \{y_1, y_2, \dots, y_{\frac{n}{2}}\}$ . Define  $c : E(\mathcal{M}(G)) \rightarrow \mathbb{Z}_3$  as follows:

For  $n \equiv 0 \pmod 4$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i-1}y_{2i-1}, y_{2i-1}x_{2i} : i \in \{1, 2, \dots, \frac{n}{4}\}\} \cup \{x'_j u : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 2 & \text{if } e \in \{y'_j u : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{y_{2i-1} : i \in \{1, 2, \dots, \frac{n}{4}\}\} \cup \{y'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 0 & \text{if } v \in \{y_{2i} : i \in \{1, 2, \dots, \frac{n}{4}\}\} \text{ or } v = u. \end{cases}$$

For  $n \equiv 2 \pmod{12}$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\} \cup \{x'_j u : j \in \{1, 2, \dots, \frac{n}{2}\}\} \cup \{y'_1 u\}, \\ 2 & \text{if } e = x_1 y'_1, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j : j \in \{2, 3, \dots, \frac{n}{2}\}\} \cup \{x'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{x_1, u\} \cup \{y_{2i} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\}, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{1, 2, \dots, \frac{n+2}{4}\}\} \cup \{y'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}. \end{cases}$$

For  $n \equiv 6 \pmod{12}$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\} \cup \{x'_j u : j \in \{1, 2, \dots, \frac{n}{2}\}\} \cup \{x_1 y'_1\}, \\ 2 & \text{if } e = y'_1 u, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{y_{2i} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\} \text{ or } v = u, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{1, 2, \dots, \frac{n+2}{4}\}\} \cup \{y'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}. \end{cases}$$

For  $n \equiv 10 \pmod{12}$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_{2i}y_{2i}, y_{2i}x_{2i+1} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\} \cup \{x'_j u : j \in \{2, 3, \dots, \frac{n}{2}\}\} \cup \{x_1 y'_1\}, \\ 2 & \text{if } e \in \{y_1 x'_1, x'_1 u, y'_1 u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{x_j, x'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}, \\ 2 & \text{if } v \in \{u, y_1\} \cup \{y_{2i} : i \in \{1, 2, \dots, \frac{n-2}{4}\}\}, \\ 0 & \text{if } v \in \{y_{2i-1} : i \in \{2, 3, \dots, \frac{n+2}{4}\}\} \cup \{y'_j : j \in \{1, 2, \dots, \frac{n}{2}\}\}. \end{cases}$$

Case 2.  $n \equiv 1 \pmod{2}$ .

Then  $P = x_1, y_1, x_2, y_2, \dots, x_{\frac{n-1}{2}} y_{\frac{n-1}{2}} x_{\frac{n+1}{2}}$ , where  $X = \{x_1, x_2, \dots, x_{\frac{n+1}{2}}\}$  and  $Y = \{y_1, y_2, \dots, y_{\frac{n-1}{2}}\}$ . Define  $c : E(\mathcal{M}(G)) \rightarrow \mathbb{Z}_3$  as follows:

For  $n \equiv 1 \pmod{4}$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{y_{2i-1}x_{2i}, x_{2i}y_{2i} : i \in \{1, 2, \dots, \frac{n-1}{4}\}\} \cup \{y'_j u : j \in \{1, 2, \dots, \frac{n-3}{2}\}\}, \\ 2 & \text{if } e \in \{x'_j u : j \in \{1, 2, \dots, \frac{n+1}{2}\}\} \cup \{x_{\frac{n+1}{2}} y'_{\frac{n-1}{2}}, y'_{\frac{n-1}{2}} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \dots, \frac{n-1}{2}\}\}, \\ 2 & \text{if } v \in \{x_{2i} : i \in \{1, 2, \dots, \frac{n-1}{4}\} \cup \{x_{\frac{n+1}{2}}\} \cup \{x'_j : j \in \{1, 2, \dots, \frac{n+1}{2}\}\}, \\ 0 & \text{if } v \in \{x_{2i-1} : i \in \{1, 2, \dots, \frac{n-1}{4}\}\} \text{ or } v = u. \end{cases}$$

For  $n \equiv 3 \pmod 4$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{y_{2i-1}x_{2i}, x_{2i}y_{2i} : i \in \{1, 2, \dots, \frac{n-3}{4}\}\} \cup \{y_{\frac{n-1}{2}}x_{\frac{n+1}{2}}\} \cup \\ & \{y'_j u : j \in \{1, 2, \dots, \frac{n-3}{2}\}\}, \\ 2 & \text{if } e \in \{x'_j u : j \in \{1, 2, \dots, \frac{n+1}{2}\}\} \cup \{x_{\frac{n+1}{2}}y'_{\frac{n-1}{2}}, y'_{\frac{n-1}{2}}u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \dots, \frac{n-1}{2}\}\}, \\ 2 & \text{if } v \in \{x_{2i} : i \in \{1, 2, \dots, \frac{n-3}{4}\} \cup \{x'_j : j \in \{1, 2, \dots, \frac{n+1}{2}\}\}, \\ 0 & \text{if } v \in \{x_{2i-1} : i \in \{1, 2, \dots, \frac{n+1}{4}\}\} \cup \{x_{\frac{n+1}{2}}\} \text{ or } v = u. \end{cases}$$

This completes the proof. □

**Corollary 2.2.** For each integer  $n \geq 3$ ,

$$\chi'_m(\mathcal{M}(P_n)) = \chi(\mathcal{M}(P_n)) = 3.$$

**Corollary 2.3.** For each integer  $n \geq 2$ ,

$$\chi'_m(\mathcal{M}(C_{2n})) = \chi(\mathcal{M}(C_{2n})) = 3.$$

**Theorem 2.4.** Let  $G$  be a bipartite graph of order 3 or more. If  $G$  contains a vertex that is adjacent to all the vertices in a partite set of  $G$ , then

$$\chi'_m(\mathcal{M}(G)) = \chi(\mathcal{M}(G)) = 3.$$

*Proof.* Let  $(X, Y)$  be the bipartition of  $G$  and let  $X = \{x_1, x_2, \dots, x_m\}$  and  $Y = \{y_1, y_2, \dots, y_n\}$ . By hypothesis,  $G$  contains a vertex, say,  $x_1$  such that  $x_1$  is adjacent to all the vertices in  $Y$ . Define  $c : E(\mathcal{M}(G)) \rightarrow \mathbb{Z}_3$  as follows:

For  $n \equiv 0 \pmod 3$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j : j \in \{4, 5, \dots, n\}\} \cup \{y'_j u : j \in \{1, 2, \dots, n\}\} \\ & \cup \{y_j x'_1 : j \in \{2, 3, \dots, n\}\}, \\ 2 & \text{if } e \in \{y_1 x'_1, x'_1 u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_2, y_3\} \cup \{y'_j : j \in \{1, 2, \dots, n\}\}, \\ 2 & \text{if } v \in \{y_1\} \cup \{y_j : j \in \{4, 5, \dots, n\}\} \cup \{u\}, \\ 0 & \text{if } v \in \{x_i, x'_i : i \in \{1, 2, \dots, m\}\}. \end{cases}$$

(For  $n = 3$ ,  $\{4, 5, \dots, n\} = \emptyset$ .)

For  $n \equiv 1 \pmod 3$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j, y_j x'_1 : j \in \{2, 3, \dots, n\}\} \cup \{y'_j u : j \in \{1, 2, \dots, n\}\} \cup \{x'_1 u\}, \\ 2 & \text{if } e = y_1 x'_1, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y'_j : j \in \{1, 2, \dots, n\}\}, \\ 2 & \text{if } v \in \{y_j : j \in \{1, 2, \dots, n\}\} \cup \{u\}, \\ 0 & \text{if } v \in \{x_i, x'_i : i \in \{1, 2, \dots, m\}\}. \end{cases}$$

For  $n \equiv 2 \pmod 3$ ,

$$c(e) = \begin{cases} 1 & \text{if } e \in \{x_1 y_j, y'_j u : j \in \{1, 2, \dots, n\}\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{y_j, y'_j : j \in \{1, 2, \dots, n\}\}, \\ 2 & \text{if } v \in \{x_1, u\}, \\ 0 & \text{if } v \in \{x_i : i \in \{2, 3, \dots, m\}\} \cup \{x'_i : i \in \{1, 2, \dots, m\}\}. \end{cases}$$

This completes the proof. □

**Corollary 2.5.** For positive integers  $r$  and  $s$  with  $r + s \geq 3$ ,

$$\chi'_m(\mathcal{M}(K_{r,s})) = \chi(\mathcal{M}(K_{r,s})) = 3.$$

Next, we consider odd cycles.

**Theorem 2.6.** For each integer  $n \geq 2$ ,

$$\chi'_m(\mathcal{M}(C_{2n+1})) = \chi(\mathcal{M}(C_{2n+1})) = 4.$$

*Proof.* Let  $C_{2n+1} = v_1 v_2 v_3 \dots v_{2n+1} v_1$ . Define  $c : E(\mathcal{M}(C_{2n+1})) \rightarrow \mathbb{Z}_4$  as follows: We consider four cases:

Case 1.  $n \equiv 0 \pmod{4}$ .

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3, 7, 11, \dots, 2n-1\} \cup \{4, 8, 12, \dots, 2n\}\} \cup \\ & \{v'_i u : i \in \{1, 3, 5, \dots, 2n+1\}\}, \\ 2 & \text{if } e \in \{v_{2n+1} v_1, v'_{2n} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{3, 5, 7, \dots, 2n-1\}\} \cup \{v'_i : i \in \{1, 3, 5, \dots, 2n+1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4, 8, 12, \dots, 2n\}\} \cup \{v_1, v'_{2n}\}, \\ 3 & \text{if } v \in \{v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2, 6, 10, \dots, 2n-2\}\} \cup \{v'_i : i \in \{2, 4, 6, \dots, 2n-2\}\}. \end{cases}$$

Case 2.  $n \equiv 1 \pmod{4}$  and  $n \geq 5$ .

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3, 7, 11, \dots, 2n-3\} \cup \{4, 8, 12, \dots, 2n-2\}\} \cup \\ & \{v'_i u : i \in \{1, 3, 5, \dots, 2n-1\}\} \cup \{v_{2n+1} v_1\}, \\ 2 & \text{if } e \in \{v_{2n-1} v_{2n}, v_{2n} v_{2n+1}, v'_{2n+1} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{1, 3, 5, \dots, 2n-3\}\} \cup \{v'_i : i \in \{1, 3, 5, \dots, 2n-1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4, 8, 12, \dots, 2n-2\}\} \cup \{v'_{2n+1}\}, \\ 3 & \text{if } v \in \{v_{2n-1}, v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2, 6, 10, \dots, 2n\}\} \cup \{v'_i : i \in \{2, 4, 6, \dots, 2n\}\}. \end{cases}$$

Case 3.  $n \equiv 2 \pmod{4}$ .

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3, 7, 11, \dots, 2n-1\} \cup \{4, 8, 12, \dots, 2n\}\} \cup \\ & \{v'_i u : i \in \{1, 3, 5, \dots, 2n+1\}\}, \\ 2 & \text{if } e \in \{v_{2n+1} v_1\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{3, 5, 7, \dots, 2n-1\}\} \cup \{v'_i : i \in \{1, 3, 5, \dots, 2n+1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4, 8, 12, \dots, 2n\}\} \cup \{v_1\}, \\ 3 & \text{if } v \in \{v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2, 6, 10, \dots, 2n-2\}\} \cup \{v'_i : i \in \{2, 4, 6, \dots, 2n\}\}. \end{cases}$$

Case 4.  $n \equiv 3 \pmod 4$ .

$$c(e) = \begin{cases} 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{3, 7, 11, \dots, 2n-3\} \cup \{4, 8, 12, \dots, 2n-2\}\} \cup \\ & \{v'_i u : i \in \{1, 3, 5, \dots, 2n-1\}\} \cup \{v_{2n+1} v_1\}, \\ 2 & \text{if } e \in \{v_{2n-1} v_{2n}, v_{2n} v_{2n+1}, v'_{2n} u, v'_{2n+1} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} 1 & \text{if } v \in \{v_i : i \in \{1, 3, 5, \dots, 2n-3\}\} \cup \{v'_i : i \in \{1, 3, 5, \dots, 2n-1\}\}, \\ 2 & \text{if } v \in \{v_i : i \in \{4, 8, 12, \dots, 2n-2\}\} \cup \{v'_{2n}, v'_{2n+1}\}, \\ 3 & \text{if } v \in \{v_{2n-1}, v_{2n+1}, u\}, \\ 0 & \text{if } v \in \{v_i : i \in \{2, 6, 10, \dots, 2n\}\} \cup \{v'_i : i \in \{2, 4, 6, \dots, 2n-2\}\}. \end{cases}$$

This completes the proof. □

Finally, we consider complete graphs.

**Theorem 2.7.** For each integer  $n \geq 3$ ,

$$\chi'_m(\mathcal{M}(K_n)) = \chi(\mathcal{M}(K_n)) = n + 1.$$

*Proof.* Let  $V(K_n) = \{v_1, v_2, \dots, v_n\}$ . Define  $c : E(\mathcal{M}(K_n)) \rightarrow \mathbb{Z}_{n+1}$  as follows:

For  $n = 2r + 1$ ,

$$c(e) = \begin{cases} i & \text{if } e = v_i v_{2r+1} \text{ for } i \in \{1, 2, \dots, 2r\}, \\ i & \text{if } e = v'_i u \text{ for } i \in \{2, 3, \dots, 2r-1\}, \\ 2r+1 & \text{if } e = v'_{2r+1} u, \\ r+1 & \text{if } e = v'_{2r} u, \\ r-1 & \text{if } e = v_{2r+1} v'_{2r}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i & \text{if } v = v_i \text{ for } i \in \{1, 2, \dots, 2r+1\}, \\ i & \text{if } v = v'_i \text{ for } i \in \{2, 3, \dots, 2r+1\}, \\ 1 & \text{if } v = u, \\ 0 & \text{if } v = v'_1. \end{cases}$$

For  $n = 4r$ ,

$$c(e) = \begin{cases} 4i - 3 & \text{if } e = v_{4i-3}v_{4i-2} \text{ for } i \in \{1, 2, \dots, r\}, \\ 1 & \text{if } e = v_{4i-2}v_{4i} \text{ for } i \in \{1, 2, \dots, r\}, \\ 4i - 1 & \text{if } e = v_{4i-1}v_{4i} \text{ for } i \in \{1, 2, \dots, r\}, \\ i & \text{if } e = v'_i u \text{ for } i \in \{1, 2, \dots, 4r\}, \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i & \text{if } v = v_i \text{ for } i \in \{1, 2, \dots, 4r\}, \\ i & \text{if } v = v'_i \text{ for } i \in \{1, 2, \dots, 4r\}, \\ 0 & \text{if } v = u. \end{cases}$$

For  $n = 4r + 2$ ,

$$c(e) = \begin{cases} i - 1 & \text{if } e = v_i v_{4r+2} \text{ for } i \in \{2, 3, \dots, 2r + 1\}, \\ i + 1 & \text{if } e = v_i v_{4r+2} \text{ for } i \in \{2r + 2, 2r + 3, \dots, 4r + 1\}, \\ i + 1 & \text{if } e = v'_i u \text{ for } i \in \{1, 2, \dots, 4r + 1\}, \\ 1 & \text{if } e \in \{v_i v_{i+1} : i \in \{1, 2, \dots, 2r\}\} \cup \{v_1 v_{2r+1}, v'_{4r+2} u\}, \\ 0 & \text{otherwise,} \end{cases}$$

and so the color sum

$$s(v) = \begin{cases} i + 1 & \text{if } v = v_i \text{ for } i \in \{1, 2, \dots, 4r + 1\}, \\ i + 1 & \text{if } v = v'_i \text{ for } i \in \{1, 2, \dots, 4r + 1\}, \\ 1 & \text{if } v = v'_{4r+2}, \\ 0 & \text{if } v \in \{v_{4r+2}, u\}. \end{cases}$$

This completes the proof. □

### 3. Conclusion

For every graph  $G$  considered in this paper, we have seen that  $\chi'_m(\mathcal{M}(G)) = \chi(\mathcal{M}(G))$ .

**Problem 3.1.** *Is it true that*

$$\chi'_m(\mathcal{M}(G)) = \chi(\mathcal{M}(G))$$

*for every connected graph  $G$  of order 3 or more?*



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