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SOME PROPERTIES OF COMAXIMAL IDEAL GRAPH OF A COMMUTATIVE RING

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Abstract. Let R be a commutative ring with identity. We use $\varphi(R)$ to denote the comaximal ideal graph. The vertices of $\varphi(R)$ are proper ideals of R which are not contained in the Jacobson radical of R, and two vertices I and J are adjacent if and only if $I + J = R$. In this paper we show some properties of this graph together with planarity of line graph associated to $\varphi(R)$.

1. Introduction

For the sake of completeness, we explain some definitions and points used throughout of the paper. A graph with vertex set V is said to be a graph on V. The vertex set of a graph G is referred to as $V(G)$, its edge set as $E(G)$. Let v be a vertex of G. The degree of $v, d(v)$, is the number of edges incident to v. An *isolated vertex* is a vertex with zero degree. The *maximum degree* of G is defined as $\Delta(G) = max\{d_G(v)|v \in G\}$. A graph with no edges is called an *empty graph*. A graph with no vertices and no edges is called a *null graph*. A vertex v of G is called a *pendant* vertex in case the degree of v is one. All pendant vertices which are adjacent to the same vertex of G together with edges is called a *horn*. A graph G is said to be *connected* if there is at least one path between every pair of vertices in G and the *distance* between two vertices v and w, $d(v, w)$, is the length of the shortest path connecting them. The *diameter* of a connected graph is the maximum of the distances between vertices. A graph in which each pair of distinct vertices is joined by an edge is called *complete graph*. We denote by K_n a complete graph with n vertices. An acyclic graph is a graph having no cycles. A connected acyclic graph is called a *tree*. Acyclic graphs are usually called $forests$. An $n-partite\ graph$ is one whose vertex set can be partitioned into n subsets so that no edge has both ends in any one subset. If G be a bipartite graph which its vertex set is partitioned into two sets X and Y and every vertex in X is joined to every vertex in Y,

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then G is called a *complete bipartite graph*. If $|X| = m$ and $|Y| = n$, we show this complete bipartite graph by $K_{m,n}$. A *star* is a complete bipartite graph with $|X| = 1$ *or* $|Y| = 1$. A graph is said to be *planar* if it can be drawn in the plane so that its edges intersect only at their ends. A subdivision of a graph is any graph that can be obtained from the original graph by replacing some edges by paths. Kuratowski's theorem says that a graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$ [2, Theorem 4.4.6]. A subgraph of G is a graph H such that $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$. A *component* of G is a maximal connected subgraph of G. A $cut-vertex$ of G, is a vertex that when removed (with its boundary edges) from G, creates a graph with more components than previously in G. The *line graph* of G (also called an interchange graph or edge graph), denoted $L(G)$, is defined by $V(L(G)) = E$ and $e_1e_2 \in E(L(G))$ if and only if e_1 and e_2 share a common vertex in G. A general prism is a polyhedron possessing two congruent polygonal faces and with all remaining faces parallelograms. An $n-prism$ *graph*, denote Y_n, and sometimes also called circular ladder graph and denoted CL_n, is a graph that has one of the prisms as its skeleton. N-prism graphs are therefore both planar and polyhedral. An n-prism graph has $2n$ vertices and $3n$ edges. A *cycle graph*, C_n, is a graph on n vertices containing a single cycle through all vertices [8].

From now on let R be a commutative ring with identity. In [6], Sharma and Bhatwadekar defined a graph on R, with vertices as elements of R, where two distinct vertices a and b are adjacent if and only if $Ra + Rb = R$.

Later, Maimani et. al. [4], studied a subgraph of the graph structure defined by Sharma and Bhatwadekar named such graph structure "*Comaximal Graphs*". They considered the subgraph of Sharma's graph, $\Gamma_2(R)$, which consists of all non-unit elements of R.

In [9], Ye and Wu defined comaximal ideal graph, $\varphi(R)$, with vertices as proper ideals of R which are not contained in the Jacobson radical of R, and two vertices I and J are adjacent if and only if $I + J = R$. Recently, in [1], the planarity and perfection of this graph were studied.

In this paper, we obtain some properties of $\varphi(R)$ and we investigate the planarity of line graph associated to $\varphi(R)$.

2. Properties of $\varphi(R)$

In this section, we investigate that complete bipartite graphs with p horns can not realizable as the graph $\varphi(R)$.

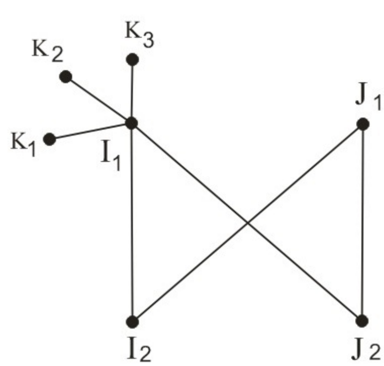

Figure 1: A complete bipartite graph together with a horn ($K_{m,n}(1)$)

We use $K_{m,n}(p)$ to denote the complete bipartite graph $K_{m,n}$ together with p horns.For example: $A = \{K_1, K_2, K_3\}$ together with the edges $I_1 - K_1, I_1 - K_2, I_1 - K_3$ is a horn at I_1 (Figure 1).

Theorem 2.1. [9, Theorem 2.4] *For a ring R, $\varphi(R)$ is a simple, connected graph with diameter less than or equal to three.*

Theorem 2.2. *Any complete bipartite graph(which is not a star) is realizable as the graph $\varphi(R)$.*

Proof. Let $G = K_{m,n}$, $m, n \geq 2$. Consider the ring $R = \mathbb{Z}_{2^m} \times \mathbb{Z}_{2^n}$ with maximal ideals $M_1 = \mathbb{Z}_{2^{m-1}} \times \mathbb{Z}_{2^n}$ and $M_2 = \mathbb{Z}_{2^m} \times \mathbb{Z}_{2^{n-1}}$. Clearly $I_i = \mathbb{Z}_{2^i} \times \mathbb{Z}_{2^n}, 0 \leq i \leq m-1$ and $J_j = \mathbb{Z}_{2^m} \times \mathbb{Z}_{2^j}, 0 \leq j \leq n-1$ are ideals contained in M_1 and M_2, respectively, which none of I_i and J_j are contained in Jacobson radical of R ($J(R) = \mathbb{Z}_2 \times \mathbb{Z}_2$). It is obvious that $V_1 = \{I_i, 0 \leq i \leq m-1\}$ and $V_2 = \{J_j, 0 \leq j \leq n-1\}$. Clearly, for each s and t, $I_s + I_t \neq R$ and $J_s + J_t \neq R$. To show that $\varphi(R) = K_{m,n}$, it is enough to prove $I_i + J_j = R$ for all $0 \leq i \leq m-1$, $0 \leq j \leq n-1$. If $I_i = \mathbb{Z}_{2^i} \times \mathbb{Z}_{2^n}$, $J_j = \mathbb{Z}_{2^m} \times \mathbb{Z}_{2^j}$ and $(x, y) \in R$, we have $x \in \mathbb{Z}_{2^m}$ and $y \in \mathbb{Z}_{2^n}$. Hence $(x, 0) \in J_j$ and $(0, y) \in I_i$ and so $(x, y) \in I_i + J_j$. Therefore $I_i + J_j = R$. □

Theorem 2.3. *Any complete bipartite graph with a horn is not realizable as the graph $\varphi(R)$.*

Proof. Let $V_1 = \{I_1, I_2, \ldots, I_m\}$ and $V_2 = \{J_1, J_2, \ldots, J_n\}$ be the two partitions. Let A be a horn at I_1 where $A = \{K_1, K_2, \ldots, K_p\}$. We have $I_i + J_j = R$ for all i, j, $I_1 + K_k = R$ for all $K_k \in A$ but $I_i + K_k \neq R$, for $i \neq 1$ and for all $K_k \in A$, $J_j + K_k \neq R$ for all $K_k \in A$. Let $i \neq 1$.

If $I_i + K_1 = I_i$, then $I_i + I_1 = I_i + I_1 + K_1 = R$, a contradiction since I_i and I_1 are not adjacent.

If $I_i + K_1 = I_1$, then $K_1 \subseteq I_1$, a contradiction since I_1 and K_1 are adjacent.

If $I_i + K_1 = K_k$, then $K_k + J_j = K_1 + I_i + J_j = R$, which contradicts the fact that K_k and J_j are not adjacent.

If $I_i + K_1 = J_j$, then $I_i \subseteq J_j$, a contradiction since I_i and J_j are adjacent.

Therefore $I_i + K_1$ for $i \neq 1$ does not exist. Hence $K_{m,n}(1)$ is not realizable as $\varphi(R)$. □

Theorem 2.4. *Any complete bipartite graph with p horns ($p \geq 2$) is not realizable as the graph $\varphi(R)$.*

Proof. $V_1 = \{I_1, I_2, \ldots, I_m\}$ and $V_2 = \{J_1, J_2, \ldots, J_n\}$ be the two partitions.

Let A and B be the two horns at I_1 and J_1 respectively where $A = \{K_1, K_2, \ldots, K_r\}$ and $B = \{L_1, L_2, \ldots, L_p\}$. We have $I_i + J_j = R,_1 + K_1 = R,_1 + L_1 = R,_i + K_1 \neq R$, $J_j + L_1 \neq R$ for all $i, j \neq 1$, $K_1 + L_1 \neq R$, $K_k + J_1 \neq R$ for all $k = 1, 2, \ldots, r$ and $L_q + I_1 \neq R$ for all $q = 1, 2, \ldots, p$.

If $K_1 + L_1 = K_k$, then $K_k + J_1 = K_1 + L_1 + J_1 = R$, a contradiction since K_k and J_1 are not adjacent.

If $K_1 + L_1 = L_q$, then $L_q + I_1 = K_1 + L_1 + I_1 = R$, which is a contradiction as L_q and I_1 are not adjacent.

If $K_1 + L_1 = I_i$, then $I_i + I_1 = K_1 + L_1 + I_1 = R$, a contradiction since I_i and I_1 are not adjacent.

If $K_1 + L_1 = J_j$, then $J_j + J_1 = K_1 + L_1 + J_1 = R$, a contradiction since J_j and J_1 are not adjacent.

Hence $K_1 + L_1$ does not exist.

Now let A and B be the two horns at I_1 and I_2 respectively where $A = \{K_1, K_2, \ldots, K_r\}$ and $B = \{L_1, L_2, \ldots, L_p\}$. It is obvious that $K_1 - I_1 - J_1 - I_2 - L_1$ is the shortest path between K_1 and L_1 which is of length 4. So $d(K_1, L_1) = 4$, which is a contradiction to the Theorem 2.1. Therefore $K_{m,n}(p)$, $p \geq 2$ is not realizable as the graph $\varphi(R)$. □

Theorem 2.5. *Let G be a triangle free graph with $diam(G) > 2$ and C_4 with two pendant vertices as a subgraph. Then G can not be realized as the graph $\varphi(R)$.*

Proof. Let $diam(G) > 2$. Let $I_1 - I_2 - I_3 - I_4 - I_1$ be a cycle of length four with two pendant J_1 and J_2 at I_1 and I_2, respectively. We have $J_1 + J_2 \neq R$.

If $J_1 + J_2 = I_1$, then $J_1 \subseteq I_1$, which is a contradiction, since I_1 and J_1 are adjacent.

If $J_1 + J_2 = I_3$, then $I_3 + I_1 = J_1 + J_2 + I_1 = R$, which is a contradiction, since I_1 and I_3 are not adjacent.

If $J_1 + J_2 = I_4$, then $I_4 + I_2 = J_1 + J_2 + I_2 = R$, which is a contradiction, since I_2 and I_4 are not adjacent.

If $J_1 + J_2 = J_1$, then $J_1 + I_2 = J_1 + J_2 + I_2 = R$, which is a contradiction, since J_1 and I_2 are not adjacent.

If $J_1 + J_2 = J_2$, then $I_1 + J_2 = I_1 + J_1 + J_2 = R$, which is a contradiction, since I_1 and J_2 are not adjacent.

Hence $J_1 + J_2$ does not exist.

Now let $J_1 + J_2 = I$. Then $J_1 \subseteq I$ and $J_2 \subseteq I$ which implies that $R = J_1 + I_1 \subseteq I + I_1$ and $R = J_2 + I_2 \subseteq I + I_2$. Hence I is common neighbour of I_1 and I_2, which is a contradiction, since G is triangle free.

If we assume that J_1 and J_2 are pendant at I_1 and I_3 or I_2 and I_4, then $d(J_1, J_2) = 4$, which is a contradiction, since $diam(\varphi(R)) \leq 3$, by Theorem 2.1. Therefore G is not realizable as the graph $\varphi(R)$. □

3. Planar line graph of $\varphi(R)$

Let $J(R)$ be Jacobson radical of R. R is said to be local if it has a unique maximal ideal. Let $Max(R)$ be the set of maximal ideals of R and $|Max(R)|$ denote the number of maximal ideals of R. For any maximal ideal M of R, $\mathcal{M}$ denotes the set of nonzero ideals contained in M and $|\mathcal{M}|$ denotes the number of ideals contained in M. In this section, we investigate the planarity of the line graph associated to the graph $\varphi(R)$. In particular, $L(\varphi(R))$ will have vertices of the form $I_{i,j}$ such that I_i and I_j are proper ideals of R which are not contained in $J(R)$ where $I_i + I_j = R$.

Theorem 3.1. *[Lemma 2.6]5 A non-empty graph G has a planar line graph $L(G)$ if and only if*

(i) G is planar,

(ii) $\Delta(G) \leq 4$,

(iii) if $deg(v) = 4$, then v is a cut-vertex in the graph G.

If $|Max(R)| = 1$, then $\varphi(R)$ is an empty graph, by [9, Proposition 2.1(1)]. Hence $L(\varphi(R))$ is a null graph. Now suppose that $|Max(R)| = r$ and $M_1, M_2, \ldots, M_r$ be distinct maximal ideals of R. Set $V_i := \mathcal{M}_i \setminus \bigcup_{j \neq i} \mathcal{M}_j$, where $1 \leq j \neq i \leq r$. It is obvious that $|V_i| \geq 1$, since $M_i \in V_i$.

Lemma 3.2. *If $L(\varphi(R))$ is planar, then $|Max(R)| \leq 4$.*

Proof. Assume to the contrary that $|Max(R)| \geq 5$ and $M_1, \ldots, M_5$ be distinct maximal ideals of R. $\varphi(R)$ contains K_5 as a subgraph. Hence $\varphi(R)$ is not planar. Therefore $L(\varphi(R))$ is not planar, by Theorem 3.1, which is a contradiction. Thereby $|Max(R)| \leq 4$. □

Now the only remaining cases for planarity of $L(\varphi(R))$ are $|Max(R)| \leq 4$.

Theorem 3.3. *Suppose that $|Max(R)| = 2$. Then $L(\varphi(R))$ is planar if and only if $|\bigcup_{i=1}^{2} V_i| \leq 5$.*

Proof. ($\Rightarrow$): Suppose that $L(\varphi(R))$ is planar and assume to the contrary that $|\bigcup_{i=1}^{2} V_i| \geq 6$. Since $|Max(R)| = 2$, $\varphi(R)$ is a complete bipartite graph, by [9, Lemma 4.1]. If $\varphi(R)$ is a star graph, then $L(\varphi(R))$ contains a subgraph isomorphic to K_5(the line graph of star graphs are complete graphs), which is not planar. Now, if $\varphi(R)$ is not a star graph, then it contains a subgraph isomorphic to $K_{2,4}$ or $K_{3,3}$. If $\varphi(R)$ contains a subgraph isomorphic to $K_{2,4}$, then $\varphi(R)$ has a vertex of degree four which is not a cut-vertex. If $\varphi(R)$ contains a subgraph isomorphic to $K_{3,3}$, then $\varphi(R)$ is not planar. In these two cases, according to Theorem 3.1, $L(\varphi(R))$ is not planar, which is a contradiction.

($\Leftarrow$): Assume that $|\bigcup_{i=1}^{2} V_i| \leq 5$. If $|\bigcup_{i=1}^{2} V_i| = 2$, then $L(\varphi(R)) \cong L(K_2) \cong K_1$. If $|\bigcup_{i=1}^{2} V_i| = 3$, then $L(\varphi(R)) \cong L(K_{1,2}) \cong K_2$. Now assume that $|\bigcup_{i=1}^{2} V_i| = 4$. Then $L(\varphi(R)) \cong L(K_{1,3}) \cong K_3$ or $L(\varphi(R)) \cong L(K_{2,2}) \cong K_{2,2}$(or $\cong C_4$) and lastly, assume that $|\bigcup_{i=1}^{2} V_i| = 5$. If $\varphi(R)$ is a star graph, then $L(\varphi(R)) \cong k_4$. Otherwise, $L(\varphi(R)) \cong L(K_{2,3}) \cong Y_3$ (Triangular prism graph) (Figure 2). □

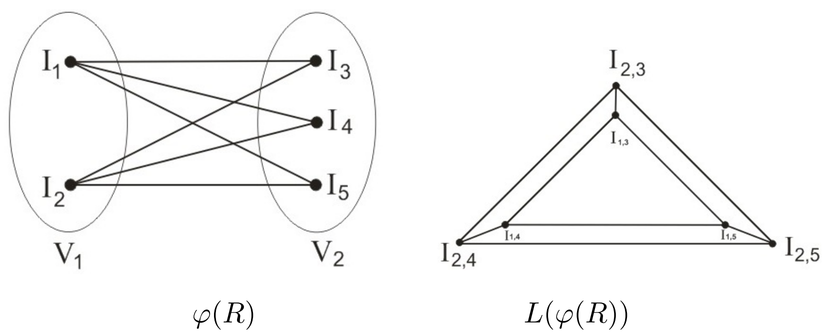

Figure 2

Assume that $|Max(R)| = 3$ and M_1, M_2 and M_3 be distinct maximal ideals of R. Set $V_i := \mathcal{M}_i \setminus \bigcup_{j \neq i} \mathcal{M}_j$, $V_{i_1 i_2} := (\mathcal{M}_{i_1} \bigcap \mathcal{M}_{i_2}) \setminus \mathcal{M}_j$ for j_1, i_2 and $1 \leq i_1 < i_2 \leq 3$.

By the above notations, we have the following lemma and theorem.

Lemma 3.4. *Suppose that $|Max(R)| = 3$. If $L(\varphi(R))$ is planar, then $|\bigcup_{i=1}^{3} V_i| \leq 4$.*

Proof. Assume to the contrary that $|\bigcup_{i=1}^{3} V_i| \geq 5$. Then $\varphi(R)$ contains a subgraph isomorphic to $K_{3,1,1}$ or $K_{2,2,1}$. Therefore $\varphi(R)$ has a vertex of degree four which is not a cut-vertex (Figure 3 and Figure 4). According to Theorem 3.1, $L(\varphi(R))$ is not planar, which is a contradiction. □

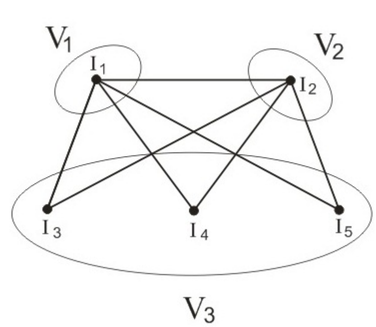

Figure 3

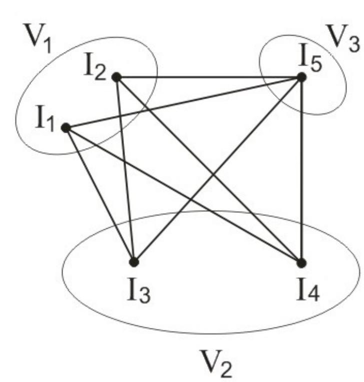

Figure 4

Theorem 3.5. *Suppose that $|Max(R)| = 3$. Then $L(\varphi(R))$ is planar if and only if one of the following conditions hold:*

(a) $|\bigcup_{i=1}^{3} V_i| = 3$ and $|V_{ij}| \leq 2$, for $1 \leq i, j \leq 3$.

(b) $|\bigcup_{i=1}^{3} V_i| = 4$ and $|V_{ij}| \leq 1$, for $1 \leq i, j \leq 3$.

Proof. (⇒): Suppose that $L(\varphi(R))$ is planar. Then $|\bigcup_{i=1}^{3} V_i| \leq 4$, by Lemma 3.4. Hence we have the following cases:

Case(1): $|\bigcup_{i=1}^{3} V_i| = 3$. For some i, j, $|V_{ij}| \geq 3$. Without loss of generality assume that $|V_{12}| = 3$, then I_3 has degree five in $\varphi(R)$ (Figure 5). Hence $L(\varphi(R))$ is not planar, by Theorem 3.1.

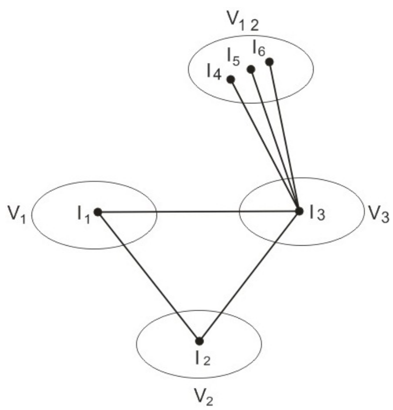

Figure 5

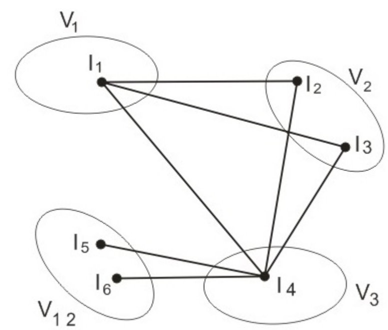

Figure 6

Case(2): $|\bigcup_{i=1}^{3} V_i| = 4$. Without loss of generality, assume that $|V_2| = 2$. If $|V_{12}|$ or $|V_{23}| \geq 2$, then $\varphi(R)$ has at least a vertex of degree five (Figure 6). Hence $L(\varphi(R))$ is not planar. If $|V_{13}| \geq 2$, then $\varphi(R)$ has a vertex of degree four which is not a cut-vertex (Figure 7).

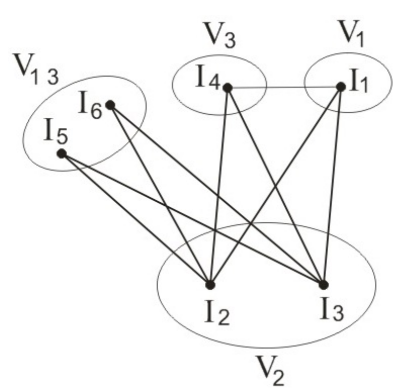

Figure 7

(⇐): Assume that $|\bigcup_{i=1}^{3} i| = 3$ and $|V_{ij}| \leq 2$. Hence the graph $\varphi(R)$ satisfies the properties of Theorem 3.1 (Figure 8). So $L(\varphi(R))$ is planar.

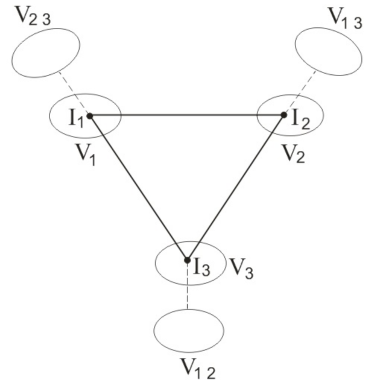

Figure 8

Now, let $|\bigcup_{i=1}^{3} V_i| = 4$ and $|V_1| = 2$, $|V_{ij}| \leq 1$. Then the graph $\varphi(R)$ satisfies the properties of Theorem 3.1 (Figure 9). Hence $L(\varphi(R))$ is planar. □

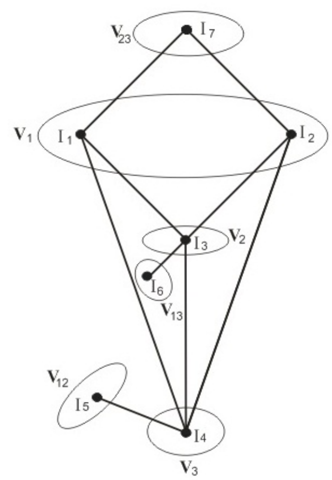

Figure 9

Lemma 3.6. *If $L(\varphi(R))$ is planar, then $|\bigcup_{i=1}^{4} V_i| = 4$.*

Proof. Assume to the contrary that $|\bigcup_{i=1}^{4} V_i| \geq 5$. Hence we have the following cases: Case(1): For only one V_i, $|V_i| \geq 2$ and $|V_j| = 1$ for all $j \neq i$, where $1 \leq i, j \leq 4$. Without loss of generality, assume that $|V_1| = 2$. Then $\varphi(R)$ has a vertex of degree four which is not a cut-vertex (Figure 10).

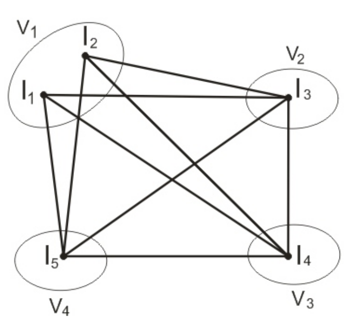

Figure 10

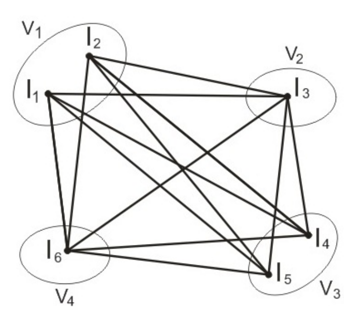

Figure 11

Case(2): For at least two V_i, $|V_i| \geq 2$. Now let without loss of generality, $|V_1| = |V_3| = 2$. Then I_1 has degree four which is not a cut-vertex or I_3 has degree five in $\varphi(R)$ (Figure 11). In these two cases, $L(\varphi(R))$ is not planar, by Theorem 3.1, which is a contradiction. □

Now, Suppose that $|Max(R)| = 4$. Set

$$V_i := \mathcal{M}_i \backslash \bigcup_{j \neq i} \mathcal{M}_j, \; V_{i_1 i_2} := (\mathcal{M}_{i_1} \bigcap \mathcal{M}_{i_2}) \backslash \bigcup_{j \neq i_1, i_2} \mathcal{M}_j,$$

$$V_{i_1 i_2 i_3} := (\mathcal{M}_{i_1} \bigcap \mathcal{M}_{i_2} \bigcap \mathcal{M}_{i_3}) \backslash \mathcal{M}_j$$

for $j \neq i_1, i_2, i_3$, $1 \leq i, j \leq 4$, where $1 \leq i_1 < i_2 < i_3 \leq 4$.

Theorem 3.7. *Suppose that $|Max(R)| = 4$. Then $L(\varphi(R))$ is planar if and only if $V_{ij} = \emptyset$ and $|V_{ijk}| \leq 1$, for all $1 \leq i, j, k \leq 4$.*

Proof. Assume that $L(\varphi(R))$ is planar. Then $|\bigcup_{i=1}^{4} V_i| = 4$, by Lemma 3.6. If $|V_{ij}| \geq 1$, for some $1 \leq i, j \leq 4$, I_4 has degree four which is not cut-vertex (Figure 12). Therefore $L(\varphi(R))$ is not planar.

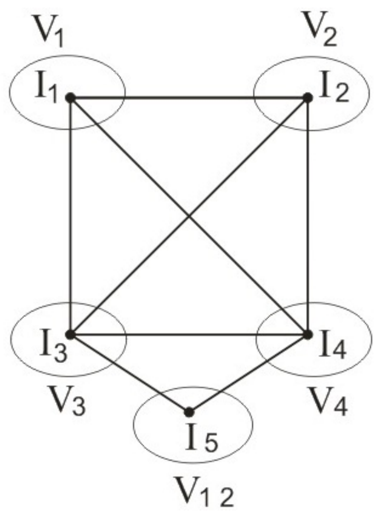

Figure 12

Now suppose that for only one V_{ijk}, $|V_{ijk}| \geq 2$, where $1 \leq i, j, k \leq 4$. Then $\varphi(R)$ has a vertex of degree five (Figure 13). Hence by Theorem 3.1, $L(\varphi(R))$ is not planar.

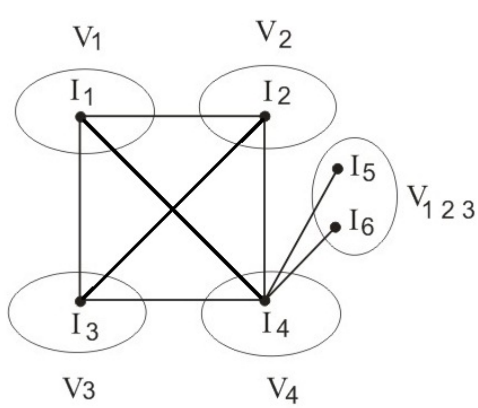

Figure 13

Conversely, suppose that $V_{12} = V_{13} = V_{23} = \emptyset$ and V_{ijk} has at most one element. Hence $\varphi(R)$ satisfies the properties of Theorem 3.1 (Figure 14) and proof is complete. □

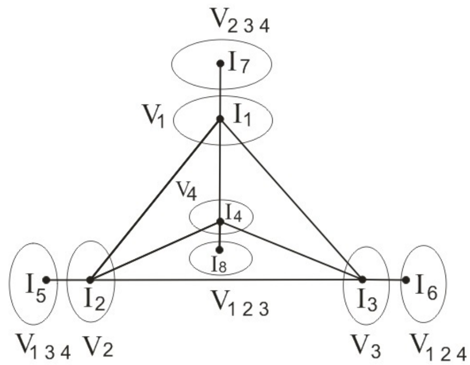

Figure 14

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