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ADJACENT VERTEX DISTINGUISHING ACYCLIC EDGE COLORING OF THE CARTESIAN PRODUCT OF GRAPHS

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Abstract. Let G be a graph and $\chi'_{aa}(G)$ denotes the minimum number of colors required for an acyclic edge coloring of G in which no two adjacent vertices are incident to edges colored with the same set of colors. We prove a general bound for $\chi'_{aa}(G\square H)$ for any two graphs G and H. We also determine exact value of this parameter for the Cartesian product of two paths, Cartesian product of a path and a cycle, Cartesian product of two trees, hypercubes. We show that $\chi'_{aa}(C_m\square C_n)$ is at most 6 fo every $m \geq 3$ and $n \geq 3$. Moreover in some cases we find the exact value of $\chi'_{aa}(C_m\square C_n)$.

1. Introduction

All of the graphs considered in this paper are simple, undirected and connected graphs. We denote by $V(G)$ and $E(G)$ the set of vertices and edges of a graph G, respectively.

An edge coloring σ is a mapping from $E(G)$ to C (C is the color set) such that any two adjacent edges receive distinct colors. An edge coloring of G is called an acyclic edge coloring if there is no 2-colored cycle in G. In other words, the subgraph of G induced by the union of any two of the colors is a forest. The acyclic chromatic index, dentoted $\chi'_a(G)$, is the least number of colors required for an acyclic edge coloring of G. The acyclic chromatic index has many practical applications such as in wavelength routing in optic networks [1].

For any vertex x of G, let $S_\sigma(x)$ denote the set of the colors of all edges incident to x. An edge coloring is said to be an adjacent vertex distinguishing edge coloring (or AVD-edge coloring) if $S_\sigma(x) \neq S_\sigma(y)$, for every adjacent vertices x and y. The minimum number of colors required for an adjacent vertex

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distinguishing edge coloring of G denote by $\chi'_{as}(G)$ and is called the adjacent vertex distinguishing edge chromatic number (or AVD-edge chromatic number) of G.

The theory of adjacent vertex distinguishing edge coloring was studied in [2, 3, 7, 8].

In this paper we study the relaxed version of this edge coloring. An acyclic edge coloring of G is called an adjacent vertex distinguishing acyclic edge coloring (or AVD-acyclic edge coloring) if it is both adjacent vertex distinguishing edge coloring and an acyclic edge coloring. The minimum number of colors required for an adjacent vertex distinguishing acyclic edge coloring of G denote by $\chi'_{aa}(G)$ and is called the adjacent vertex distinguishing acyclic edge chromatic (or AVD-acyclic edge chromatic number) of G. An AVD-acyclic edge coloring of G using k colors is denoted by k-AVD-acyclic edge coloring.

Zhang et al. [5] studied this coloring and determined $\chi'_{aa}(G)$ for many basic families of graphs, including cycles, trees, fans, wheels, some complete bipartite graphs and some complete graphs. They proposed the following conjecture.

Conjecture 1.1. *For every connected graph G of order at least 3, if $G \neq C_5$ and $G \neq K_n$ (the complete graph of order n), then $\chi'_{aa}(G) \leq \Delta(G) + 2$.*

Liu et al. [4] proved for any graph G without isolated edges, $\chi'_{aa}(G) \leq 32\Delta(G)$. Also in [5], was proved the following theorem:

Theorem 1.2. *Suppose that G is a graph with $\Delta(G) \geq 10^{20}$. If the girth $g(G) \geq 222\Delta(G)\log\Delta(G)$, then $\chi'_{aa}(G) \leq \Delta(G) + 301$.*

Notice that an AVD-acyclic edge coloring of a graph G is an AVD-edge coloring of G, therefore $\chi'_{aa}(G) \geq \chi'_{as}(G)$. Also we have the following proposition:

Proposition 1.3. *For every graph G, $\chi'_{aa}(G) \geq \Delta(G)$. Moreover every graph G with two adjacent vertices of degree $\Delta(G)$ satisfies $\chi'_{aa}(G) \geq \Delta(G) + 1$.*

The following lemmas will be used in the this paper. Let P_n and C_n denote the path and the cycle on n vertices, respectively.

Lemma 1.4. [5] *For any $n \geq 3$,*

$$\chi'_{as}(P_n) = \chi'_{aa}(P_n) = \begin{cases} 2 & n = 3; \\ 3 & n \geq 4. \end{cases}$$

Lemma 1.5. [5] *For any $n \geq 3$,*

$$\chi'_{as}(C_n) = \chi'_{aa}(C_n) = \begin{cases} 3 & n \equiv 0 (\mathrm{mod} 3); \\ 4 & n \not\equiv 0 (\mathrm{mod} 3), n \neq 5; \\ 5 & n = 5. \end{cases}$$

Lemma 1.6. [5] *For any tree T of order $n \geq 3$, without two vertices of maximum degree, we have $\chi'_{as}(T) = \chi'_{aa}(T) = \Delta(T)$. Otherwise $\chi'_{as}(T) = \chi'_{aa}(T) = \Delta(T) + 1$.*

The Cartesian product of two graphs G and H denoted by $G\Box H$, is a graph with vertex set $V(G)\times V(H)$, where (u,v) is adjacent to (u',v') when either $u=u'$ and $vv'\in E(H)$ or $v=v'$ and $uu'\in E(G)$. For a vertex $v\in V(H)$, let $G_v=G\Box\{v\}$ and for a vertex $u\in V(G)$, let $H_u=\{u\}\Box H$.

In Section 2, we give an upper bound for $\chi'_{aa}(G\Box H)$ and determine exact of $\chi'_{aa}(G\Box H)$ for some graphs G and H, espacially the Cartesian product of two trees.

In Section 3, we determine the exact values of $\chi'_{aa}(G\Box P_n)(n\geq 4)$, $\chi'_{aa}(P_m\Box P_n)$, $\chi'_{aa}(P_m\Box C_n)$ and $\chi'_{aa}(Q_d)$.

In Section 4, we give an upper bound for $\chi'_{aa}(C_m\Box C_n)$ and for any $m=3k,4k,5k$ and n, $(m,n)\neq(4,4)$, we show $\chi'_{aa}(C_m\Box C_n)=5$.

2. AVD-acyclic edge coloring $G\Box H$

In this section we give an upper bound for AVD-acyclic edge chromatic number of the Cartesian product of graphs, which is a generalization of the upper bound for AVD-edge chromatic number of the Cartesian product of graphs in [7]. An important corollary of this theorem will be used to prove Conjecture 1.1 for some Cartesian product of graphs. The following lemma will be used for stating the theorem.

Lemma 2.1. [7] *Let $\mathcal{C}=\{0,1,\ldots,k-1\}, k\geq 2$, and let π^l be a permutation of $\mathcal{C}$ such that $\pi^l(c)=(c+l)(\mathrm{mod}k)$ for every $c\in\mathcal{C}$, where l is a positive integer. We have*

(i) *the minimum l for which $\pi^l(\mathcal{A})=\mathcal{A}$ for every non-empty proper subset $\mathcal{A}$ of $\mathcal{C}$, is equal to the minimum prime factor of k,*

(ii) *if t is relatively prime with k (that is, $(t,k)=1$), then the minimum l for which $\pi^l(\mathcal{A})=\mathcal{A}$ for every non-empty proper subset $\mathcal{A}$ consisting of t elements of $\mathcal{C}$, is equal to k.*

Theorem 2.2. *Let G be a graph having a vertex coloring with r colors and an acyclic edge coloring with $k\geq\Delta(G)+1$ colors. Let H be a graph having an AVD-acyclic edge coloring with $k'\geq\Delta(H)+1$ colors. Then*

$$\chi'_{aa}(G\Box H)\leq k+k'-1$$

if one of the following two conditions holds:

(i) *the minimum prime factor of k' is at least r,*

(ii) *H is a t-regular graph satisfying $k'\geq r$ and $(t,k')=1$.*

Proof. The vertex set of G can be partitioned into r independent sets $V_0,V_1,\ldots,V_{r-1}$. Let σ_H be an AVD-acyclic edge coloring of H with k' colors $0,1,\ldots,k'-1$ and for every $e\in E(H)$,

$$\sigma^i_H(e)=(\sigma_H(e)+i)(\mathrm{mod}k'),$$

where $i=0,1,\ldots,r-1$. Let also σ_G be an acyclic edge coloring of G with k colors distinct from those used by σ_H.

In order to obtain an AVD-acyclic edge coloring σ of $G\Box H$, we first use the coloring σ_G to color each copy of G in $G\Box H$ and for each vertex $a\in V_i$, use the coloring σ^i_H to color the copy H_a of in $G\Box H$, where $i=0,1,\ldots,r-1$. Note that there exists at least one color j such that $j\notin S_{\sigma_G}(a)$. Then each vertex of H_a misses the color j. So we modify the coloring σ_H^i of H_a by changing the color 0 by the color j.

The length of any 2-colored cycle can be appear in $G\Box H$ is four, but Lemma 2.1 and Conditions (i), (ii) guarantee, there is no such 2-colored cycle in $G\Box H$. So the obtained edge coloring is acyclic. The proof of there is no pair of adjacent vertices meet the same set of colors is the same as for the case AVD-edge coloring of Theorem 2.1 in [7]. □

Corollary 2.3. *Let G be a graph having a vertex coloring with k' colors and acyclic edge coloring with $k \geq \Delta(G)+1$ colors. Let H be a graph having an AVD-acyclic edge coloring with $k' \geq \Delta(H)+1$ colors, where k' is a prime. Then*

$$\chi'_{aa}(G\Box H) \leq k + k' - 1.$$

Theorem 2.4. *Let G_i be a t_i-regular graph, where $i = 1, 2, \ldots, d$ and $t_1 \geq t_2 \geq \cdots \geq t_d$. If $\chi'_{aa}(G_1) = t_1 + 1$ and G_i, $(i = 2, 3, \ldots, d)$ having a $(t_i + 1)$-acyclic edge coloring, then for any integer $d \geq 2$, we have*

$$\chi'_{aa}(G_1\Box G_2\Box \cdots \Box G_d) = \sum_{i=1}^{d} t_i + 1.$$

Proof. We prove the theorem by induction on d. Note that $\chi'_{aa}(G_1) = t_1 + 1$, $(t_1, t_1 + 1) = 1$ and the chromatic number of G_2 is at most $t_1 + 1$ (because $t_1 \geq t_2$). When $d = 2$, by Theorem 2.2, we have $\chi'_{aa}(G_1\Box G_2) \leq t_1 + t_2 + 1$. Assume that

$$\chi'_{aa}(G_1\Box G_2\Box \cdots \Box G_{d-1}) = \sum_{i=1}^{d-1} t_i + 1.$$

Let $G' = G_1\Box G_2\Box \cdots \Box G_{d-1}$ and $t = \sum_{i=1}^{d-1} t_i$. Note that G' is a t-regular graph, so the chromatic number of G_d is at most $t + 1$ (because $t_d \leq t$) and $(t, t + 1) = 1$. By Theorem 2.2, we have

$$\chi'_{aa}(G'\Box G_d) \leq t_d + t + 1 = \sum_{i=1}^{d} t_i + 1.$$

Since $G_d\Box G'$ is a $(t+t_d)$-regular graph, by Proposition 1.3 we have $\chi'_{aa}(G'\Box G_d) \geq \sum_{i=1}^{d} t_i+1$. Therefore,

$$\chi'_{aa}(G_1\Box G_2\Box \cdots \Box G_d) = \sum_{i=1}^{d} t_i + 1.$$

□

Theorem 2.5. *Let G and H be two trees of order at least 3. If both G and H have no two adjacent vertices of maximum degree, then $\chi'_{aa}(G\Box H) = \Delta(G) + \Delta(H)$, otherwise,*

$$\chi'_{aa}(G\Box H) = \Delta(G) + \Delta(H) + 1.$$

Proof. Assume that both of G and H have no two adjacent vertices of maximum degree. By Proposition 1.3, $\chi'_{aa}(G\Box H) \geq \Delta(G) + \Delta(H)$. Let σ be an AVD-acyclic edge coloring of G with $\Delta(G)$ colors and σ' be an AVD-acyclic edge coloring of H with $\Delta(H)$ colors distinct from the colors used by σ.

Let V_0 and V_1 be a partition of $V(G)$ to independent sets. In order to obtain an AVD-acyclic edge coloring π of $G\square H$, we first color each copy of G as σ and for each vertex $a \in V_0$ color the copy H_a as σ'. For each vertex $a \in V_1$ color the copy H_a as $\sigma' + 1$ where addition is taken modulo $\Delta(H)$.

It is easy to check that π is an AVD-acyclic edge coloring of $G\square H$ with $\Delta(G) + \Delta(H)$ colors. So $\chi'_{aa}(G\square H) \leq \Delta(G) + \Delta(H)$. Therefore $\chi'_{aa}(G\square H) = \Delta(G) + \Delta(H)$.

Now assume that G or H have two adjacent vertices of maximum degree. By Proposition 1.3, $\chi'_{aa}(G\square H) \geq \Delta(G) + \Delta(H) + 1$. Also by Lemma 1.6 and Theorem 2.2, we have $\chi'_{aa}(G\square H) \leq \Delta(G) + \Delta(H) + 1$. So $\chi'_{aa}(G\square H) = \Delta(G) + \Delta(H) + 1$. □

3. AVD-acyclic edge coloring $G\square P_n$

Note that $G\square P_n$ has two vertices of maximum degree for any $n \geq 4$, so by Proposition 1.3, $\chi'_{aa}(G\square P_n) \geq \Delta(G) + 3$. Also if G have a vertex coloring with 3 colors and an acyclic edge coloring with $\Delta(G) + 1$ colors, then by Corollary 2.3 and Lemma 1.4, for any $n \geq 4$ we have $\chi'_{aa}(G\square P_n) \leq \Delta(G) + 3$. Hence we have the following lemma:

Lemma 3.1. *If G be a graph having a vertex coloring with 3 colors and having an acyclic edge coloring with $\Delta(G) + 1$ colors, then for any $n \geq 4$, we have $\chi'_{aa}(G\square P_n) = \Delta(G) + 3$.*

Theorem 3.2. *Let G be a graph having an AVD-acyclic edge coloring with $k \geq \Delta(G) + 1$ colors, then $\chi'_{aa}(G\square P_2) \leq k + 1$.*

Proof. The graph $G' = G\square P_2$ consist of two copies G_0 and G_1 of G and a perfect matching between them. Take an AVD-acyclic edge coloring of G, say σ with colors $0, 1, \cdots, k-1$. We now define an acyclic edge coloring σ' of G' as follows: for an edge $e \in E(G')$,

$$\sigma'(e) = \begin{cases} \sigma(e) & e \in G_0; \\ (\sigma(e) + 1)(\text{mod} k) & e \in G_1; \\ k & o.w. \end{cases}$$

It is easy to see that σ' is an AVD-acyclic edge coloring of G' with $k+1$ colors. So $\chi'_{aa}(G') \leq k+1$. □

Corollary 3.3. *For every $m \geq n \geq 2$,*

$$\chi'_{aa}(P_m\square P_n) = \begin{cases} 5 & m \geq 3, n \geq 4; \\ 4 & o.w. \end{cases}$$

Proof. Let $m = n = 2$, then by Lemma 1.5 we can obtain an AVD-acyclic edge coloring with 4 colors of $P_2\square P_2 = C_4$. For $m = 2$ and $n \geq 3$, by Theorem 3.2 and for $m, n \geq 3$ by Theorem 2.5 we provide an AVD-acyclic edge coloring of $P_m\square P_n$. □

Notice that the d-dimensional hypercube Q_d is the Cartesian product of P_2 by itself d times, we can obtain a theorem as follows:

Corollary 3.4. *For any $d \geq 3$, we have $\chi'_{aa}(Q_d) = d + 1$.*

Proof. For $d = 3$, we can easily get a 4-AVD-acyclic edge coloring of Q_3 (Figure 1). Let $\chi'_{aa}(Q_{d-1}) = d$. Since $Q_d = Q_{d-1}\square P_2$, then by Theorem 3.2, we obtain $\chi'_{aa}(Q_d) \leq d + 1$. Also by Proposition 1.3, $\chi'_{aa}(Q_d) \geq d + 1$. So $\chi'_{aa}(Q_d) = d + 1$.

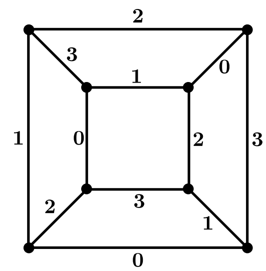


FIGURE 1. A 4-AVD-acyclic edge coloring of Q_3.

□

Corollary 3.5. *For any integers $m_1 \geq 4$ and $m_i \geq 3$, $i = 2, \ldots, d$,*

$$\chi'_{aa}(P_{m_1}\square P_{m_2}\square\cdots\square P_{m_d}) = 2d + 1.$$

Proof. By Corollary 3.3, $\chi'_{aa}(P_{m_1}\square P_{m_2}) = 5$. Assume that

$$\chi'_{aa}(P_{m_1}\square P_{m_2}\cdots\square P_{m_{d-1}}) = 2d - 1.$$

Let $G = P_{m_1}\square P_{m_2}\cdots\square P_{m_{d-1}}$. So by Theorem 2.2, $\chi'_{aa}(G\square P_{m_d}) \leq 2d + 1$.

Also by Proposition 1.3, $\chi'_{aa}(G\square P_{m_d}) \geq 2d + 1$. Hence,

$$\chi'_{aa}(P_{m_1}\square P_{m_2}\square\cdots\square P_{m_d}) = \chi'_{aa}(G\square P_{m_d}) = 2d + 1.$$

□

Let A and B are edge colorings of $C_m\square C_n(P_m\square C_n)$ and $C_m\square C_{n'}(P_m\square C_{n'})$, respectively. We shall denote by $A + B$ the pattern $A|B$ of size $m \times (n + n')$ obtained by "gluing" together the pattern A and B. Moreover, we shall denote by lA, $l \geq 2$, the pattern of size $m \times ln$ obtained by gluing together l copies of the pattern A. If A and B to be AVD-acyclic edge colorings, then lA is an AVD-acyclic edge coloring. Whenever it is not necessary the pattern $A + B$ to be an AVD-acyclic edge coloring. In the following we provide the patterns A and B, such that $A + B$ to be an AVD-acyclic edge coloring.

Proposition 3.6. *For every $m, n \geq 3$, $p, q \geq 1$, if $\chi'_{aa}(C_m\square C_n) \leq k$, then $\chi_{aa}(C_{pm}\square C_{qn}) \leq k$.*

To see that, it is enough to observe that every AVD-acyclic edge coloring σ of $C_m\square C_n$ can be extended to an AVD-acyclic edge coloring of $C_{pm}\square C_{qn}$ by repeating the pattern given by σ, p times vertically and q times horizontally.

Let x and y be two integers, and

$$S(x, y) = \{\alpha x + \beta y | \alpha, \beta \text{ are nonnegative integers}\}.$$

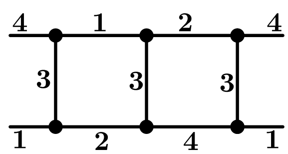
Sylvester has shown the following lemma.

Lemma 3.7. [6] *Let x and y be relatively prime integers greater than 1. Then $n \in S(x,y)$ for all $n \geq (x-1)(y-1)$.*

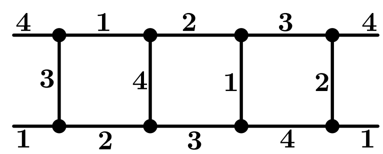
Lemma 3.8. *For every $m \geq 2$, $n \geq 3$, we have:*

$$\chi'_{aa}(P_m \Box C_n) = \begin{cases} 4 & m = 2, n \neq 5; \\ 5 & o.w. \end{cases}$$

Proof. Let $m = 2$. We consider the following patterns, where provide a 4-AVD-acyclic edge coloring of $P_2 \Box C_3$ and $P_2 \Box C_4$, respectively.



Pattern A



Pattern B

Let $n = 3\alpha + 4\beta$. We use the combination of patterns A and B as the pattern $\alpha A + \beta B$ to get a 4-AVD-acyclic edge coloring of $P_2 \Box C_n$ for every $n \in S(3,4)$. So for every $n \geq 3$ and $n \neq 5$, we have $\chi'_{aa}(P_2 \Box C_n) = 4$.

For $n = 5$, since $\chi'_{as}(P_2 \Box C_5) = 5$ (see [3]), so $\chi'_{aa}(P_2 \Box C_5) \geq 5$. In the Figure 2, we present a 5-AVD-acyclic edge coloring of $P_2 \Box C_5$.

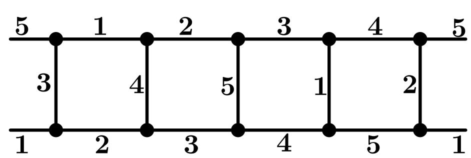
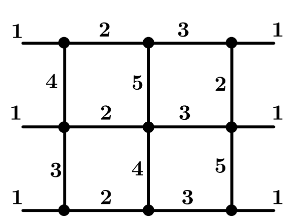
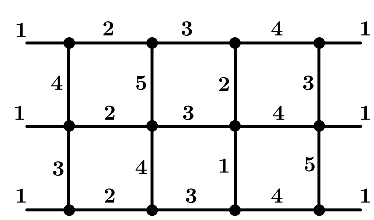


FIGURE 2. A 5-AVD-acyclic edge coloring of $P_2 \Box C_5$.

Assume now $m = 3$. We use the following patterns C and D. It is easy to check that this patterns provide a 5-AVD-acyclic edge coloring of $P_3 \Box C_3$ and $P_3 \Box C_4$, respectively.



Pattern C



Pattern D

Let $n = 3\alpha + 4\beta$. We use the combination of patterns C and D as the pattern $\alpha C + \beta D$ to get a 5-AVD-acyclic edge coloring of $P_3 \Box C_n$ for any $n \in S(3,4)$. So for every $n \geq 3$ and $n \neq 5$, we have $\chi'_{aa}(P_3 \Box C_n) = 5$.

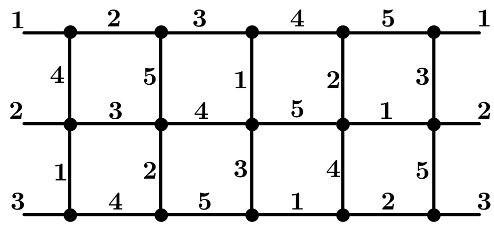


FIGURE 3. A 5-AVD-acyclic edge coloring of $P_3 \square C_5$.

For $n = 5$, the pattern was given in Figure 3 provides a 5-AVD-acyclic edge coloring of $P_3 \square C_5$. Clearly for any $m \geq 4$, we can provide a 5-AVD acyclic edge coloring of $P_m \square C_n$ by Lemma 3.1. $\square$

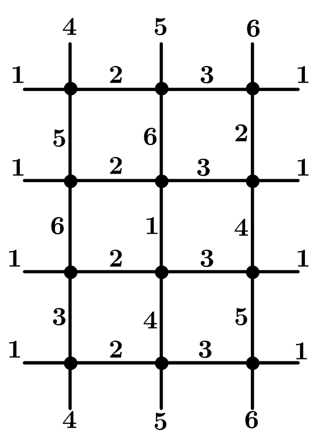
4. AVD-acyclic edge coloring $G \square C_k$

In this section first we give an upper bound for $\chi'_{aa}(C_m \square C_n)$ and determine the exact value of $\chi'_{aa}(C_m \square C_n)$ for some cases.

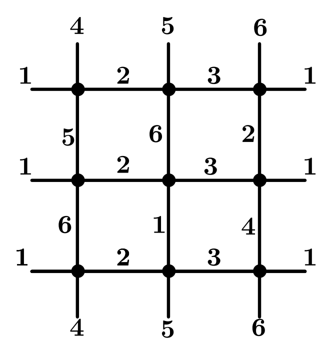
Theorem 4.1. *For every $m, n \geq 3$, we have $\chi'_{aa}(C_m \square C_n) \leq 6$.*

Proof. If m or n to be even numbers, then by Theorem 2.2 and Lemma 1.5, $\chi'_{aa}(C_m \square C_n) \leq 6$.

Now let m and n be odd numbers. Cleary, the following Patterns E and F are 6-AVD-acyclic edge colorings of $C_4 \square C_3$ and $C_3 \square C_3$, respectively.



Pattern E

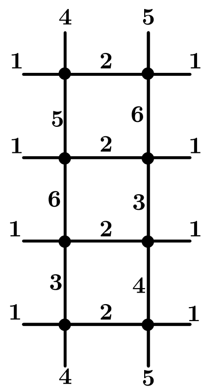


Pattern F

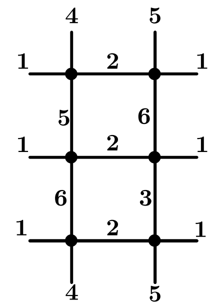
Also by considering

E	G
F	H

we obtain a 6-AVD-acyclic edge coloring of $C_7 \square C_5$, where blocks G and H are the followings:



Block G



Block H

By an appropriate combination of these patterns E, F and blocks G, H we can obtain a 6-AVD-acyclic edge coloring of $C_m \square C_n$, where $m = 2\alpha + 3\beta$ and $n = 2\alpha' + 3\beta'$. So for any $m \in S(2,3)$ and $n \in S(2,3)$, we have $\chi'_{aa}(C_m \square C_n) \leq 6$.

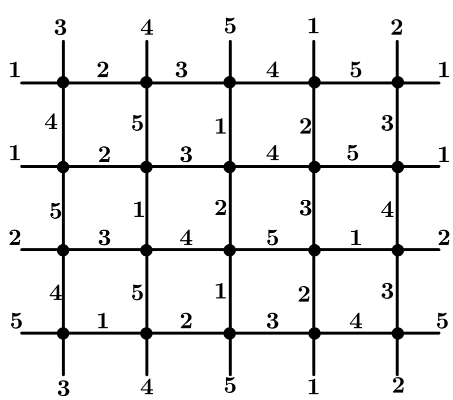
□

Theorem 4.2. *For every k and $n \geq 3$, we have:*

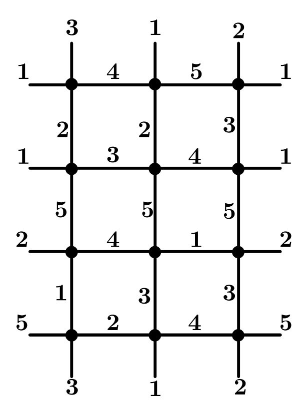
1. $\chi'_{aa}(C_{3k} \square C_n) = 5$,
2. $\chi'_{aa}(C_{4k} \square C_n) = 5$, *except* $(k, n) = (1, 4)$,
3. $\chi'_{aa}(C_{5k} \square C_n) = 5$,

Proof. By Proposition 3.6, it is sufficient to consider $k = 1$.

1. For every k and $n \geq 3$, by Theorem 2.2 and Lemma 1.5 we have $\chi'_{aa}(C_3 \square C_n) = 5$.
2. The following patterns are 5-AVD-acyclic colorings of $C_4 \square C_5$ and $C_4 \square C_3$, respectively.



Pattern I



Pattern J

Let $n = 5\alpha + 3\beta$, by an appropriate combination of these patterns as the pattern $\alpha I + \beta J$ we can obtain a 5-AVD-acyclic edge coloring of $C_4 \square C_n$ where $n \in S(5,3)$. So for any $n \geq 3$ and $n \neq 4, 7$, we have a 5-AVD-acyclic edge coloring of $C_4 \square C_n$.

For $C_4 \square C_7$, in the Figure 4, a 5-AVD-acyclic edge coloring of it is given.

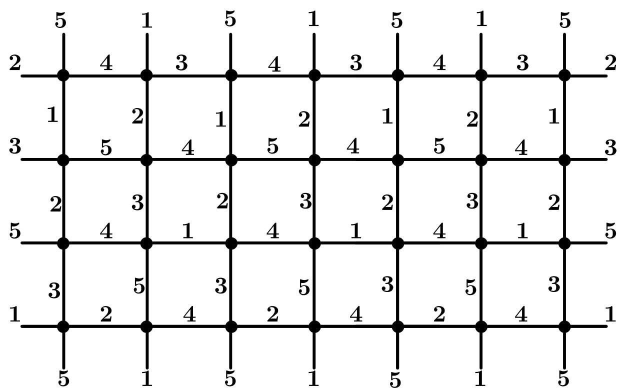
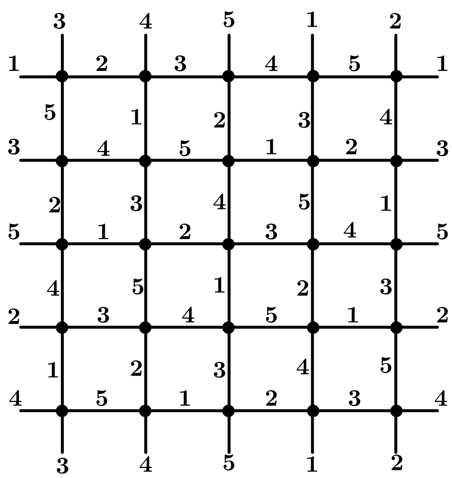
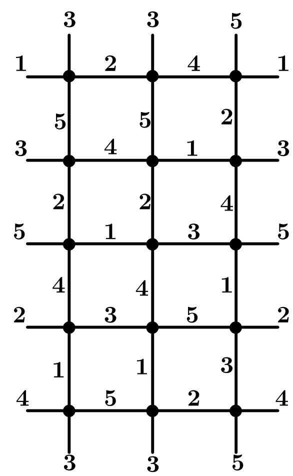


FIGURE 4. A 5-AVD-acyclic edge coloring of $C_4 \Box C_7$.

3. In the following, the patterns K and L provide a 5-AVD-acyclic edge coloring of $C_5 \Box C_5$ and $C_5 \Box C_3$, respectively.



Pattern K



Pattern L

Let $n = 5\alpha + 3\beta$, by an appropriate combination of these patterns as the pattern $\alpha K + \beta L$ we can obtain a 5-AVD-acyclic edge coloring of $C_5 \Box C_n$ where $n \in S(5,3)$. So for any $n \geq 3$ and $n \neq 4, 6, 7$, we have $\chi'_{aa}(C_5 \Box C_n) = 5$.

The cases $n = 4, 6$ was done in the items 1 and 2 of peresent theorem. A 5-AVD-acyclic edge coloring of $C_5 \Box C_7$ is given in Figure 5.

□

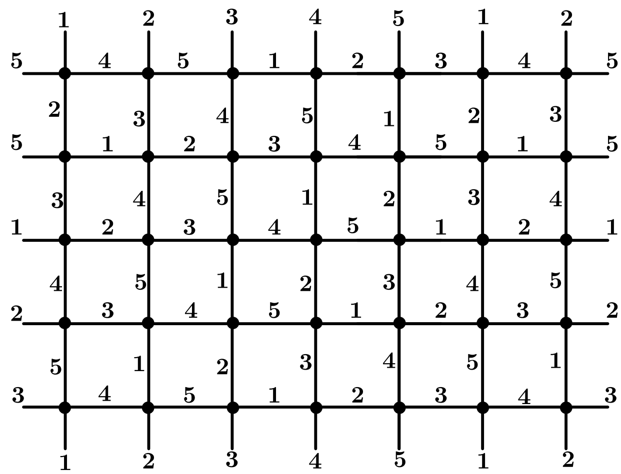


FIGURE 5. A 5-AVD-acyclic edge coloring of $C_5 \Box C_7$.

For any $m = 3k, 4k$ or $5k$ and n, $(m,n) \neq (4,4)$ we have, $\chi'_{aa}(C_m \Box C_n) = 5$. Let $G_1 = C_m \Box C_n$ and $G_i = C_{m_{i+1}}$, $i = 2, 3, \ldots, d-1$. By Theorem 2.4, we can obtain the following corollary:

Corollary 4.3. *For any $m_1 = 3k, 4k$ or $5k$ $(m_1 \neq 4)$ and m_i, $i = 2, \ldots, d$, we have*

$$\chi'_{aa}(C_{m_1} \Box C_{m_2} \Box \cdots \Box C_{m_d}) = 2d + 1.$$

By the above results we make the following conjecture:

Conjecture 4.4. *For any $m \geq 3$ and $n \geq 3$, we have $\chi'_{aa}(C_m \Box C_n) = 5$.*

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