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THE REFORMULATED SOMBOR INDEX OF A GRAPH

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ABSTRACT. In 2021, Gutman invented a novel degree-based topological index called the Sombor index, inspired by a geometric interpretation of degree-radii of the edges and invited researchers to investigate their mathematical properties and chemical meanings. The Sombor index was reformulated in terms of the edge degree instead of the vertex degree as the original Sombor Index. In this paper, we compute the exact values of a certain class of graphs. Also, some bounds in terms of the order, size, minimum/maximum degrees and other topological indices are obtained.

1. Introduction

Let $G = (V, E)$ be a non-trivial, simple and undirected with vertex set $V = V(G)$ and edge set $E = E(G)$. Also, $|V| = n$ and $|E| = m$ denotes the number of vertices and number of edges in G. The degree $deg_G(u)$ or $d_G(u)$ of a vertex $u \in V(G)$ is the number of vertices adjacent to u. The maximum degree and minimum degree are denoted by $\Delta = \Delta(G)$ and $\delta = \delta(G)$, respectively. The degree of an edge $e = uv \in E(G)$ is the number of edges incident to e. As well known, $d_G(e) = d_G(u) + d_G(v) - 2$ and $e \sim f$ means that the edges e and f are adjacent, i.e., they share a common end-vertex in G. For graph-theoretic terminology not defined here, we follow [17].

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In 2004, Milicevic [25], the first and second reformulated Zagreb indices in terms of edge degrees instead of vertex degrees are defined as

$$EM_1(G) = \sum_{e\sim f} \left[d_G(e) + d_G(f)\right], \text{ and} \tag{1.1}$$

$$EM_2(G) = \sum_{e\sim f} d_G(e)\, d_G(f). \tag{1.2}$$

For more details on reformulated Zagreb indices and their relaetd topological indices, we refer to [20, 27, 29, 38, 39].

In 2021, Gutman [13], invented a novel degree-based topological index, called the Sombor index, inspired by a geometric interpretation of degree-radii of the edges. The Sombor index of a graph G is defined as

$$SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}\,. \tag{1.3}$$

It attracted much attention from scholars, and its mathematical properties and chemical applicability were, and currently is, much investigated, see for instance [6, 7, 9, 10, 11, 12, 15, 23, 24, 28, 31, 33, 34, 35, 36, 40, 41].

Analogously, we introduced the edge-degree-based topological index instead of the vertex-degree-based topological index called reformulated Sombor (RS) index of a graph G and is defined as

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}. \tag{1.4}$$

For any adjacency edges $e, f \in E(G)$ with $e = uv$ and $f = vw$, then an equation (1.4), can be written as

$$RS(G) = \sum_{e\sim f} \left[(d_G(u) + d_G(v) - 2)^2 + (d_G(v) + d_G(w) - 2)^2\right]^{\frac{1}{2}}.$$

$$RS(G) = \sum_{e\sim f} \Big[(d_G(u) + d_G(v))^2 - 4(d_G(u) + d_G(v) + 4 + (d_G(v) + d_G(w))^2$$

$$-4(d_G(v) + d_G(w)) + 4\Big]^{\frac{1}{2}}. \tag{1.5}$$

The Line graph $L(G)$ is the graph with vertex set $V(L(G)) = E(G)$ and whose vertices correspond to the edges of a graph G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common, see [18].

Observation 1.1. *As $L(G)$ is defined on the edge set of a graph G, isolated vertices of G if any plays no role in $L(G)$. Clearly, $RS(G) = SO(L(G))$.*

Observation 1.2. *For any non-trivial connected graph G,*

$$2\delta - 2 \le d_G(uv) \le 2\Delta - 2 \text{ for every edge } uv \in G. \tag{1.6}$$

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2. Bounds in terms of minimum/maximum degree and size

For obtaining some bound on $RS(G)$, we make use of the definition of Platt number and its preliminary lemma's as follows:
The Platt number is the sum of degrees of all its edges of G and is denoted as

$$\begin{aligned} Pl(G) &= \sum_{uv\in E(G)} d_G(uv) \\ &= \sum_{uv\in E(G)} [d_G(u)+d_G(v)-2] \end{aligned}$$

$$= \sum_{u\in V(G)} d_G(u)(d_G(u)-1). \tag{2.1}$$

This concept was introduced by Platt [30] in 1952 and he was focused in design a scheme for predicting the physicochemical (paraffin) properties of organic compounds. For more details, we refer to [1, 2, 8].

Observation 2.1. *The hyper platt number $Pl_2(G) = \sum_{uv\in E(G)}[d_G(uv)]^2$ coincides with the first reformulated Zagreb index (i.e., equation (1.1)).*

Lemma 2.1. [3] *For any connected graph G with $n \geq 2$,*

(i) $2m\,(\delta-1) \leq Pl(G) \leq 2m\,(\Delta-1)$.
(ii) $m \leq Pl(G) \leq 2m\,(n-2)$.

Lemma 2.2. [1, 14] *For any graph G with $n \geq 2$, $Pl(G) = M_1(G) - 2m$.*

Theorem 2.1. *Let G be a (n,m)-connected graph with $n \geq 3$. Then,*

$$2\sqrt{2}\,m\,(\delta-1)^2 \leq RS(G) \leq 2\sqrt{2}\,m\,(\Delta-1)^2. \tag{2.2}$$

Further, equality holds if and only if G is regular.

Proof. Let G be a (n,m)-connected graph with $n \geq 3$. For any edge $e \in E(G)$.
By equation (i) of Lemma 2.1 and equation (1.5), we have

$$2\sqrt{2}(\delta-1) \leq \sqrt{d_G(e)^2+d_G(f)^2} \leq 2\sqrt{2}(\Delta-1). \tag{2.3}$$

The equation (2.3) satisfies for $\frac{Pl(G)}{2}$ pairs of adjacent edges and sum of those inequalities results.

$$Pl(G)\,\sqrt{2}(\delta-1) \leq \sum_{e\sim f}\sqrt{d_G(e)^2+d_G(f)^2} \leq Pl(G)\,\sqrt{2}(\Delta-1). \tag{2.4}$$

Again, by equation (i) of Lemma 2.1 and equation (2.4), we have

$$2\sqrt{2}\,m\,(\delta-1)^2 \leq RS(G) \leq 2\sqrt{2}\,m\,(\Delta-1)^2.$$

Further equality holds if and only if G is a regular. □

3. Bounds in terms of order and size

Theorem 3.1. *Let G be a (n, m)-connected graph. Then,*

$$\frac{m}{\sqrt{2}} \leq RS(G) \leq 2\sqrt{2}\, m\,(n-2)^2.$$

Proof. Let G be a (n, m)-connected graph with $n \geq 3$, for any edge $e \in E(G)$,

$$1 \leq d_G(e) \leq 2(n-2). \tag{3.1}$$

Using equation (ii) of Lemma 2.1, we have

$$\frac{m}{2} \leq \frac{Pl(G)}{2} \leq m\,(n-2).$$

Therefore, $1 \leq \{d_G(e)^2, d_G(f)^2\} \leq 4(n-2)^2$ implies that

$$\sqrt{2} \leq \sqrt{d_G(e)^2 + d_G(f)^2} \leq 2\sqrt{2}(n-2).$$

The above equation satisfies for $\frac{Pl(G)}{2}$ pairs of adjacent edges and the sum of those inequalities results

$$\frac{Pl(G)}{\sqrt{2}} \leq \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2} \leq Pl(G)\sqrt{2}(n-2). \tag{3.2}$$

Again by equation (ii) of Lemma 2.1 and equation (3.2), we have

$$\frac{m}{\sqrt{2}} \leq \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2} \leq 2\,\sqrt{2}\, m\,(n-2)^2.$$

Therefore, $\frac{m}{\sqrt{2}} \leq RS(G) \leq 2\,\sqrt{2}\, m\,(n-2)^2$. □

4. Bounds in terms of first and second Zagreb indices

In 1972, Gutman and Trinajstic [16] introduced the first and second Zagreb indices of G are defined as

$$M_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad M_2(G) = \sum_{uv\in E(G)} d_G(u)\, d_G(v).$$

By Lemma 2.2 and equation (3.2), we have

Theorem 4.1. *Let G be a (n, m)-connected graph. Then,*

(i) $\frac{M_1(G)-2m}{\sqrt{2}} \leq RS(G) \leq (M_1(G) - 2m)\sqrt{2}(n-2)$.
(ii) $(\sqrt{2}M_1(G) - 2\sqrt{2}m)(\delta - 1) \leq RS(G) \leq (\sqrt{2}M_1(G) - 2\sqrt{2}m)(\Delta - 1)$.

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5. Bounds in terms of reformulated Zagreb Indices

Theorem 5.1. *Let G be a (n,m)-connected graph. Then,*

$$\frac{EM_1(G)}{2} \leq RS(G) \leq EM_1(G).$$

The equality holds if and only if G is regular.

Proof. Let G be a (n,m)-connected graph. Then,

$$\frac{d_G(e)+d_G(f)}{2} \leq \sqrt{d_G(e)^2+d_G(f)^2} \leq d_G(e)+d_G(f).$$

The above inequality satisfies for each edges e and f such that $e, f \in E(G)$ and taking the sum on all those inequalities we have,

$$\sum_{e\sim f} \frac{d_G(e)+d_G(f)}{2} \leq \sum_{e\sim f} \sqrt{d_G(e)^2+d_G(f)^2} \leq \sum_{e\sim f} \left[d_G(e)+d_G(f)\right].$$

Therefore,

$$\frac{EM_1(G)}{2} \leq RS(G) \leq EM_1(G).$$

The above inequality equality holds when G is a regular. □

Theorem 5.2. *Let G be a (n,m)-connected graph with $\delta \geq 2$. Then,*

$$\frac{EM_2(G)}{(\Delta-1)} \leq RS(G) \leq \frac{EM_2(G)}{(\delta-1)}.$$

The equality holds if and only if G is regular.

Proof. Let G be a (n,m)-connected graph with $\delta \geq 2$. Then,

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2+d_G(f)^2}.$$

$$= d_G(e)d_G(f)\left(\sqrt{\frac{1}{d_G(e)^2}+\frac{1}{d_G(f)^2}}\right).$$

$$\sqrt{\frac{1}{2(\Delta-1)^2}+\frac{1}{2(\Delta-1)^2}} \leq \sqrt{\frac{1}{d_G(e)^2}+\frac{1}{d_G(f)^2}} \leq \sqrt{\frac{1}{2(\delta-1)^2}+\frac{1}{2(\delta-1)^2}}.$$

$$\frac{1}{(\Delta-1)} \leq \sqrt{\frac{1}{d_G(e)^2}+\frac{1}{d_G(f)^2}} \leq \frac{1}{(\delta-1)}.$$

$$\frac{d_G(e).d_G(f)}{(\Delta-1)} \leq \sqrt{d_G(e)^2+d_G(f)^2} \leq \frac{d_G(e).d_G(f)}{(\delta-1)}.$$

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The above inequality satisfies for each edges $e, f \in E(G)$ and taking the summation of all those inequalities, we have

$$\sum_{e\sim f}\frac{d_G(e).d_G(f)}{(\Delta-1)} \leq \sum_{e\sim f}\sqrt{d_G(e)^2+d_G(f)^2} \leq \sum_{e\sim f}\frac{d_G(e).d_G(f)}{(\delta-1)}.$$

$$\frac{EM_2(G)}{(\Delta-1)} \leq RS(G) \leq \frac{EM_2(G)}{(\delta-1)}.$$

The above inequality equality holds when G is regular. □

6. Bounds in terms of reformulated first Zagreb index and Inverse sum edge indeg index

In 2016, Bhanumathi et al., [4] introduced the edge version of Inverse sum indeg of G and is defined as

$$ISI_e(G) = \sum_{e\sim f}\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}.$$

Theorem 6.1. *Let G be a (n, m)-connected graph. Then,*

$$2\sqrt{2}ISI_e(G) \leq RS(G) \leq \sqrt{2}\big[EM_1(G) - 2ISI_e(G)\big].$$

The equality hold if and only if G is regular.

Proof. Let G be a (n, m)-connected graph. Then,

$$RS(G) = \sum_{e\sim f}\sqrt{d_G(e)^2+d_G(f)^2}.$$

$$2\sqrt{2}\Big[\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}\Big] \leq \sqrt{d_G(e)^2+d_G(f)^2} \leq \sqrt{2}\Big[\frac{d_G(e)^2+d_G(f)^2}{d_G(e)+d_G(f)}\Big].$$

$$2\sqrt{2}\Big(\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}\Big) \leq \sqrt{d_G(e)^2+d_G(f)^2} \leq \sqrt{2}\big(d_G(e)+d_G(f)\big) - 2\sqrt{2}\Big(\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}\Big).$$

The above inequality, which satisfies for each edge $e, f \in E(G)$ and taking the sum of all those inequalities, we have

$$2\sqrt{2}\sum_{e\sim f}\Big(\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}\Big) \leq \sum_{e\sim f}\sqrt{d_G(e)^2+d_G(f)^2} \leq \sqrt{2}\sum_{e\sim f}\Big[\big(d_G(e)+d_G(f)\big) - 2\Big(\frac{d_G(e).d_G(f)}{d_G(e)+d_G(f)}\Big)\Big].$$

$$2\sqrt{2}ISI_e(G) \leq RS(G) \leq \sqrt{2}\Big[EM_1(G) - 2ISI_e(G)\Big].$$

The above inequality equality holds when G is a regular. □

Theorem 6.2. *Let G be a (n, m)-connected graph. Then,*

$$\frac{2\sqrt{2}}{3}\big[EM_1(G) - ISI_e(G)\big] \leq RS(G) \leq \sqrt{2}\big[EM_1(G) - 2ISI_e(G)\big].$$

The inequality equality holds if and only if G is regular.

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Proof. Let G be a (n, m)-connected graph. Then,

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}.$$

$$\frac{2\sqrt{2}}{3}\Big[(d_G(e) + d_G(f)) - \frac{d_G(e)d_G(f)}{d_G(e) + d_G(f)}\Big] \leq \sqrt{d_G(e)^2 + d_G(f)^2}$$
$$\leq \Big[\sqrt{2}(d_G(e) + d_G(f)) - 2\sqrt{2}\Big(\frac{d_G(e)d_G(f)}{d_G(e) + d_G(f)}\Big)\Big].$$

The above inequality which satisfies for each edges $e, f \in E(G)$ and taking the sum of all those inequalities, we have

$$\frac{2\sqrt{2}}{3}\sum_{e\sim f}\Big[(d_G(e) + d_G(f)) - \frac{d_G(e)d_G(f)}{d_G(e) + d_G(f)}\Big] \leq \sum_{e\sim f}\sqrt{d_G(e)^2 + d_G(f)^2}$$
$$\leq \sqrt{2}\sum_{e\sim f}\Big[(d_G(e) + d_G(f)) - 2\Big(\frac{d_G(e)d_G(f)}{d_G(e) + d_G(f)}\Big)\Big].$$

Therefore,

$$\frac{2\sqrt{2}}{3}\big[EM_1(G) - ISI_e(G)\big] \leq RS(G) \leq \sqrt{2}\big[EM_1(G) - 2ISI_e(G)\big].$$

The above inequality equality holds when G is regular. □

7. Bounds in terms of Randic edge index

In 1975, Milan Randic [32] introduced the Randic index of a graph G. In 1998 Bollobas and Erdos [5] introduced the General Randic index, and in 2017, [37] H. Yang et al., introduced the edge version of General Randic index, and is defined as

$$ER_\alpha = \sum_{e\sim f}\big(d_G(e).d_G(f)\big)^\alpha.$$

For $\alpha = -1, -\frac{1}{2}, \frac{1}{2}, 1$.

Theorem 7.1. *Let G be a (n, m)-connected graph. Then,*

$$ER_{-\frac{1}{2}}(G)2\sqrt{2}(\delta - 1)^2 \leq RS(G) \leq 2\sqrt{2}(\Delta - 1)^2\, ER_{-\frac{1}{2}}(G).$$

The equality holds if and only if G is regular.

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Proof. Let G be a (n, m)-connected graph. Then,

$$\begin{aligned} RS(G) &= \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}. \\ &= \sum_{e\sim f} \frac{1}{\sqrt{d_G(e)d_G(f)}} \sqrt{d_G(e)d_G(f)\Big[d_G(e)^2 + d_G(f)^2\Big]}. \\ &\leq \sum_{e\sim f} \frac{1}{\sqrt{d_G(e)d_G(f)}} \sqrt{4(\Delta-1)^2 2(\Delta-1)^2}. \\ &\leq \sum_{e\sim f} \frac{1}{\sqrt{d_G(e)d_G(f)}} 2\sqrt{2}(\Delta-1)^2. \\ &\leq \ ER_{-\frac{1}{2}}(G) 2\sqrt{2}(\Delta-1)^2. \end{aligned}$$

Similarly,

$$\begin{aligned} RS(G) &= \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}. \\ &\geq \ ER_{-\frac{1}{2}}(G) 2\sqrt{2}(\delta-1)^2. \end{aligned}$$

Therefore,

$$ER_{-\frac{1}{2}}(G) 2\sqrt{2}(\delta-1)^2 \leq RS(G) \leq \ ER_{-\frac{1}{2}}(G) 2\sqrt{2}(\Delta-1)^2.$$

The above inequality equality holds if and only if G is regular. □

Theorem 7.2. *Let G be a (n, m)-connected graph. Then*

$$\sqrt{2}\, ER_{\frac{1}{2}}(G) \leq RS(G).$$

The equality holds if and only if G is regular.

Proof. Let G be a (n, m)-connected graph. Then,

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}.$$

$$\sqrt{2}\sqrt{d_G(e)d_G(f)} \leq \sqrt{d_G(e)^2 + d_G(f)^2}.$$

The above inequality, which satisfies for each edges $e, f \in E(G)$ and taking the sum of all those inequalities we have,

$$\sqrt{2}\sum_{e\sim f} \sqrt{d_G(e)d_G(f)} \leq \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}.$$

Therefore,

$$\sqrt{2}\, ER_{\frac{1}{2}}(G) \leq RS(G).$$

The above inequality equality holds when G is regular. □

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8. Bounds in terms of first Forgotten edge index

In 2017, Kulli [21] introduced the edge version of the Forgotten index of a graph G and is defined as

$$EF_1(G) = \sum_{e\sim f} d_G(e)^2 + d_G(f)^2.$$

Theorem 8.1. *Let G be a (n, m)-connected graph with $\delta \geq 2$. Then,*

$$\frac{EF_1(G)}{2\sqrt{2}(\Delta - 1)} \leq RS(G) \leq \frac{EF_1(G)}{2\sqrt{2}(\delta - 1)}.$$

The equality holds if and only if G is regular.

Proof. Let G be a (n, m)-connected graph $\delta \geq 2$. Then,

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}.$$

$$= \sum_{e\sim f} \frac{d_G(e)^2 + d_G(f)^2}{\sqrt{d_G(e)^2 + d_G(f)^2}}.$$

$$\leq \sum_{e\sim f} \left[d_G(e)^2 + d_G(f)^2\right] \frac{1}{\sqrt{(2(\delta-1))^2 + (2(\delta-1))^2}}.$$

$$\leq \frac{EF_1(G)}{2\sqrt{2}(\delta - 1)}.$$

Similarly,

$$RS(G) = \sum_{e\sim f} \sqrt{d_G(e)^2 + d_G(f)^2}.$$

$$\geq \frac{EF_1(G)}{2\sqrt{2}(\Delta - 1)}.$$

Therefore,

$$\frac{EF_1(G)}{2\sqrt{2}(\Delta - 1)} \leq RS(G) \leq \frac{EF_1(G)}{2\sqrt{2}(\delta - 1)}.$$

The above inequality equality holds if and only if when G is regular. □

9. Specific families of graphs

Since G is an r- regular graph (i.e., $\delta = \Delta = r$). By equation (2.2), we have the following result without proof.

Theorem 9.1. *For any r- regular graph G with $n \geq 3$ and $r \geq 1$,*

$$RS(G) = 2\sqrt{2}m(r-1)^2. \quad (9.1)$$

In view of equations (1.5) and (9.1), we arrive at.

Theorem 9.2. (i) *For any complete K_n with $n \geq 3$ and $r = n-1$,*

$$RS(K_n) = 2\sqrt{2}\, m\, (n-2)^2.$$

(ii) *For any cycle C_n with $n \geq 3$ and $r = 2$,*

$$RS(C_n) = 2\sqrt{2}\, n.$$

(iii) *For any complete bipartite graph $K_{r,s}$ with $1 \leq r \leq s$,*

$$RS(K_{r,s}) = \frac{r.s}{2}(s+r-2)[\sqrt{2}\,(r+s-2)].$$

(iv) *For any wheel W_n with $n \geq 4$,*

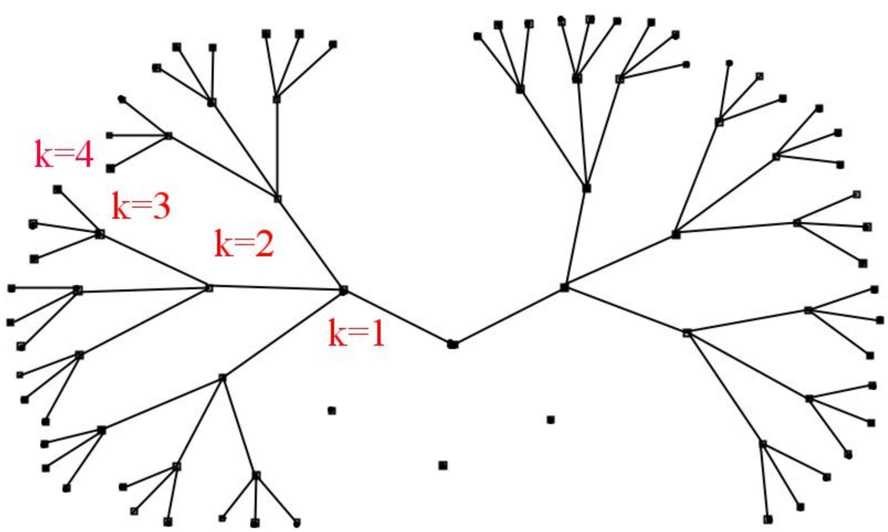
$$RS(W_n) = \frac{n(n-1)(n-2)}{\sqrt{2}} + 4\sqrt{2}(n-1) + 2(n-1)\sqrt{n^2+16}.$$

(v) *For any path P_n with $n \geq 2$,*

$$RS(P_n) = 2\sqrt{5} + 2\sqrt{2}(n-4).$$

10. **Chemical graphs: Regular dendrimers**

Dendrimers are highly ordered, branched polymeric molecules and symmetric about the core and often have a spherical three-dimensional morphology. Dendrimers have a very well-defined chemical structure with three major building parts. These are the center, branches and end groups, where new branches are radiated from the center and are added in advance steps, for more information we refer to [19, 22, 26].



Regular Dendrimer: $T_{k,s}$

The Regular dendrimer $T_{k,s}$, where k denotes the growth (radius if $s > 1$) and s represent the degree of the central vertex, except central and pendent vertices the degree the a vertex is 4. If $s = 1$ then the radius and growth value is not same so we neglet the case. When the degree of central vertex $s > 1$ and the growth is $k = 1$, then the graph is a Tree. If $k = 2$, we computed the values manually. When the value of $s \geq 1$ and $k \geq 3$ then the degree of a edge set and the corresponding edge partitions as follows.

(i) Number of vertices is $s\left(\frac{3^{k-1}}{2}\right) + 1$, out of which 1 vertex of degree s, $s\left(\frac{3^{k}-1}{2}\right)$ vertices of degree 4 and $s\ 3^{k-1}$ are pendent.

(ii) Number of edges is $s\left(\frac{3^{k}-1}{2}\right)$, in which s - edges of degree $s+1$, $s\ 3^{k-1}$ vertices of degree 3 and $3\ s\ \left(\frac{3^{k-2}}{2}\right)$ is of degree 6.

(iii) Number of pairs of adjacent edges is $s\ (3^{k} - 3 + \frac{s-1}{2})$, based on degree of the edges e and f we partitioned as follows.

TABLE 1. The edge partitions based on degree of adjacent edges.

Adjacent edges						
	$d_G(e)$	3	3	6	6	$s+1$
	$d_G(f)$	3	6	6	$s+1$	$s+1$
Number of pairs		$s\ 3^{k-1}$	$s\ 3^{k-1}$	$s(3^{k-1}-6)$	$3\ s$	$\frac{s(s-1)}{2}$

TABLE 2. The Computed values in orbitrary values of $s > 1$ and $k > 2$.

Indices	Computed values
$Pl(T_{k,s})$	$s(s+2+2(3)^{k})$
$RS(T_{k,s})$	$3^{k}s(3\sqrt{2}+\sqrt{5}) + s\left(\frac{s^2-1}{\sqrt{2}} + 3\sqrt{s^2+2s+37} - 36\sqrt{2}\right)$
$EM_1(T_{k,s})$	$s\left(3^{k+2} + s^2 + 3s - 52\right)$
$EM_2(T_{k,s})$	$21s3^{k} + s\left(\frac{(s^2-1)(s+1)}{2} + 18s - 198\right)$
$ISI_e(T_{k,s})$	$s3^{k-1}(\frac{13}{2}) + s\left(\frac{s^2}{4} + \frac{18(s+1)}{s+7} - \frac{73}{4}\right)$
$ER_{\frac{1}{2}}(T_{k,s})$	$s3^{k}\left(3+\sqrt{2}\right) + s\left(3\sqrt{6(s+1)} + \frac{s^2-1}{2} - 36\right)$
$ER_{-\frac{1}{2}}(T_{k,s})$	$s3^{k-2}\left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right) + s\left(\frac{s-1}{2(s+1)} + \frac{3}{\sqrt{6(s+1)}} - 1\right)$
$EF_1(T_{k,s})$	$45s3^{k} + s\left((s-1)(s+1)^2 + 3s^2 + 6s - 321\right)$

TABLE 3. The Particular values in orbitrary values of s and k.

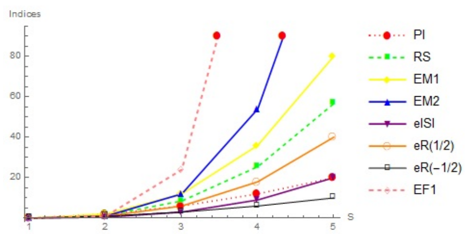
Indices	s	$k=1$	$k=2$	$k=3$	$k=4$
$Pl(T_{k,s})$	1	0	11	48	156
	2	2	24	98	314
	3	6	39	177	501
	4	12	56	204	636
	5	20	75	260	830
$RS(T_{k,s})$	1	0	20.665	118.681	493.988
	2	1.414	29.269	296.950	996.650
	3	8.485	111.354	453.909	1503.46
	4	25.455	153.755	632.203	2031.604
	5	42.426	263.380	832.199	2581.450
$EM_1(T_{k,s})$	1	0	36	162	684
	2	2	100	410	1382
	3	12	204	627	2085
	4	36	360	876	2820
	5	80	580	1155	3585
$EM_2(T_{k,s})$	1	0	45	297	1539
	2	1	117	862	3130
	3	12	210	1317	4719
	4	54	438	1914	6450
	5	160	765	2655	8325
$ISI_e(T_{k,s})$	1	0	5	37.5	163.5
	2	0.5	14.5	97.4	331.4
	3	3	30	149.1	500.1
	4	9	53	209.727	677.727
	5	20	85	277.5	862.5
$ER_{\frac{1}{2}}(T_{k,s})$	1	0	16.348	77.911	334.279
	2	1	39	199.761	676.496
	3	6	70.176	305.641	1020.743
	4	18	112.475	428.461	1381.931
	5	40	168.639	565.918	1757.756
$ER_{-\frac{1}{2}}(T_{k,s})$	1	0	2.224	5.828	19.571
	2	1	4.333	12.717	39.202
	3	3	6.348	19.421	59.178
	4	6	8.298	25.876	78.846
	5	10	10.202	32.273	98.486
$EF_1(T_{k,s})$	1	0	100	702	3348
	2	1	239	1910	6770
	3	24	456	2913	10203
	4	162	802	4164	13884
	5	640	1340	5715	17865

11. Graphical Comparision

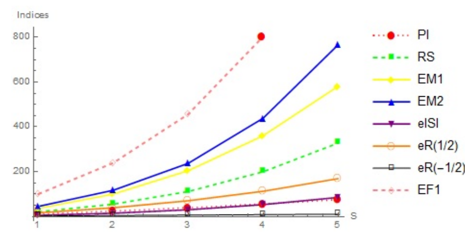
Topological indices of dendrimers are useful in theoretical chemistry, pharmacology, toxicology, and environmental chemistry. In this paper we compute the Platt index, the reformulated Sombor index, the reformulated first and second Zagreb indices, the edge version of the Inverse sum indeg, the edge version of the

Randic index, and the edge version of the Forgotten index. From $k = 1$ to $k = 4$, shows that the edge version of Forgotten index gets the highest value compared to all other topological indices, and the edge version of the Randic index gives the least value. When k grows, the remaining index values change. We can represent this mathematically as

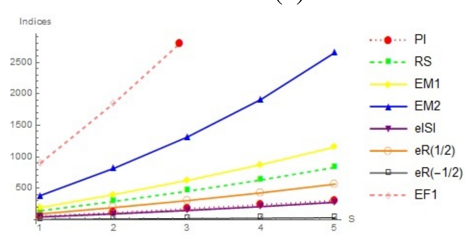
$$ER_{-\frac{1}{2}} <_e ISI < Pl < ER_{\frac{1}{2}} < RS < EM_1 < EM_2 < EF_1.$$



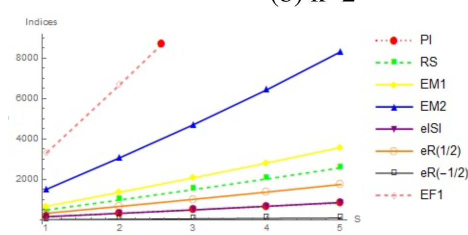
(a) k=1



(b) k=2



(c) k=3



(d) k=4

12. **Conclusion**

Being new edge (bond) degree-based topological index of a graph, the reformulated Sombor index and its properties are relatively unknown. In this article, we compute exact values for various classical graphs and found some bounds in terms of order, size, and degree. Also, were compared to various edge degree-based topological indices. Finally, we computed these indices for the Regular dendrimer and compared them graphically. Further, the comparative advantages, applications and mathematical point of view, many questions are suggested by this research, among them are the following.

1. Find the extremal values and extremal graphs of the reformulated Sombor index. Also, characterize this edge version sombor index in terms of other edge degree-based topological indices.
2. Find the values of the reformulated Sombor index of all classes of chemical graphs and compare with other edge degree - based topological indices. Also, explore some results towards QSPR / QSAR Model.

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References

[1] A. Alir and D. Dimitrov, On the extremal graphs with respect to bond incident degree indices, *Discrete Appl. Math.*, **238** (2018) 32–40.

[2] M. Barysz, D. Plavsic and N. Trinajstic, A note on the Topological indices, *MATCH Commun. Math. Comput. Chem.*, **19** (1986) 89–116.

[3] M. M. Belavadi and T. A. Mangam, Platt number of total graphs, *International Journal of Applied Mathematics*, **31)** (2018).

[4] M. Bhanumathi, K. Easu Julia Rani and S. Balachandran, The edge version of inverse sum indeg index connected graph, *International Journal of Mathematical Archive*, **7** (2016) 8–12.

[5] B. Bhanumathi and P. Erdos, Graphs of extremal weights, *Ars comb.*, **50** (1998) 225.

[6] R. Cruz, I. Gutman and J. Rada, Sombor index of chemical graphs, *Appl. Math. Comput.*, **399** (2021) 126018.

[7] K. C. Das, A. S. Cevik, I. N. Cangul and Y. Shang, On Sombor index, *Symmetry*, **13** (2021) 140.

[8] J. Devillers and A. T. Balaban and (Eds.), *Topological Indices and Related Descriptors in QSAR and QSPR*, Gordon and Breach, Amsterdam, (1999).

[9] T. Došlic, T. Réti, and A. Ali. On the structure of graphs with integer Sombor indices, *Discrete Math. Lett.*, **7** (2021) 1–4.

[10] S. Filipovski, Relations between Sombor index and some topological indices, *Iran. J. Math. Chem.*, **12** (2021) 19–26.

[11] Y. Gao, W. Sajjad, A. Q. Baig and M. R. Farahani, The edge version of randic, zagreb, atom bond connectivity and geometric-arithmetic indices of $HAC_5C_6C_7[P,Q]$ nanotube, *International Journal of Pure and Applied Mathematics*, **115** (2017) 405–418.

[12] N. Ghanbari, On the Sombor characteristic polynomial and Sombor energy of a graph, *Comput. Appl. Math. Journal Profile*, **41** (2022) 1–14.

[13] I. Gutman, Geometric approach to degree-based topological indices: Sombor indices, *MATCH Commun. Math. Comput. Chem.*, **86** (2021) 11–16.

[14] I. Gutman and E. Estrada, Topological indices based on the line graph of the molecular graph, *Journal of chemical information and computer sciences*, **36** (1996) 541–543.

[15] I. Gutman and K. Ch. Das, The first Zagreb index 30 years after, *MATCH Commun. Math. Comput. Chem.*, **50** (2004) 83–92.

[16] I. Gutman and N. Trinajstic, Graph theory and molecular orbitals. Total ϕ-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, **17** (1972) 535–538.

[17] F. Harary, *Graph theory*, Addison-Wesley, Reading Mass, (1969).

[18] R. L. Hemminger and L. W. Beineke, Line graphs and line digraphs, Selected Topics in Graph Theory, *Acad. Press Inc.*, (1978) 271–305.

[19] S. M. Kang, M. A. Zahid, W. Nazeer and W. Gao, Calculating the degree based topological indices of dendrimers, *Open Chemistry*, **16** (2018) 681–688.

[20] V. R. Kulli, *Edge version of inverse sum indeg index of certain nanotubes and nanotori*, **7** (2017) 855–858.

[21] V. R. Kulli, Edge version of F-index, general sum connectivity index of certain nanotubes, *Annals of Pure and Applied Mathematics*, **14** (2017) 449–455.

[22] V. R. Kulli, *Multiplicative Connectivity Indices of Nanostructures*, Lap lembert Academic Publishing, (2018) 1–10.

[23] V. R. Kulli, Sombor index of certain graph operators, *Int. J. Eng. Sci. Res. Technol.*, **10** (2021) 127–134.

[24] H. Liu, I. Gutman, L. You and Y. Huang, Sombor index: Review of extremal results and bounds, *J. Math. Chem.*, **66** (2022) 771–798.

[25] A. Milicevic, S. Nikolic and N. Trinajstic, On reformulated Zagreb indices, *Molecular diversity*, **8** (2004) 393–399.

[26] G. R. Newkome, N. C. Moorefield and F. Vogtle, *Dendrimers and dendrons: concepts, syntheses, applications*, Weinheim: Wiley-vch, **632** (2001).

[27] N. De, Some bounds of reformulated Zagreb indices, *Appl. Math. Sci.*, **6** (2012) 5005–5012.

[28] M. R. Oboudi, On graphs with integer Sombor index, *Journal of Applied Mathematics and Computing*, (2022) 1–12.

[29] K. Pattabiraman and A. Santhakumar, Bounds on first reformulated Zagreb index of graph, *Casp. J. Math. Sci*, **7** (2018) 25–35.

[30] J. R. Platt, Prediction of isomeric differences in paraffin properties, *J. Phys. Chem.*, **56** (1952) 328–336.

[31] J. Rada, J. M. Rodriguez and J. M. Sigarreta, General properties on Sombor indices, *Discrete Appl. Math.*, **299** (2021) 87–97.

[32] M. Randic, Characterization of molecular branching, *Journal of the American Chemical Society*, **97** (1975) 6609–6615.

[33] I. Redzepovic, Chemical applicability of Sombor indices, *J. Serb. Chem. Soc.*, **86** (2021) 445–457.

[34] T. Reti, T. Doslic and A. Ali, On the Sombor index of graphs, *Contrib. Math.*, **3** (2021) 11–18.

[35] Y. Shang, Sombor index and degree-related properties of simplicial networks, *Appl. Math. Comput.*, **419** (2022) 126881.

[36] Z. Wang, Y. Mao, Y. Li and B. Furtula, On relations between Sombor and other degree-based indices, *J. Appl. Math. Comput.*, **68** (2022) 1–17.

[37] H. Yang, W. Sajjad, A. Q. Baig and M. R. Farahani, The Edge Version of Randic, Zagreb, Atom Bond Connectivity and Geometric-Arithmetic Indices of NA_Q^P Nanotube, *International journal of advanced biotechnology and research*, **8** (2017) 1582–1589.

[38] S. Zafar, R. Nazir, M. S. Sardar and Z. Zahid, *Edge version of harmonic index and harmonic polynomial of some classes of graphs*, textbf34 (2017) 479–486.

[39] B. Zhou and N. Trinajstic, Some properties of the reformulated Zagreb indices, *Journal of mathematical chemistry*, **48** (2010) 714–719.

[40] B. Zhou, Zagreb indices, *MATCH Commun. Math. Comput. Chem.*, **52** (2004).

[41] T. Zhou, Z. Lin and L. Miao, *The Sombor index of trees and unicyclic graphs with given maximum degree*, arXiv preprint arXiv, (2021).

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