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CONVOLUTIONAL CYLINDER-TYPE BLOCK-CIRCULANT CYCLE CODES

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Abstract. In this paper, we consider a class of column-weight two quasi-cyclic low-density parity check codes in which the girth can be large enough, as an arbitrary multiple of 8. Then we devote a convolutional form to these codes, such that their generator matrix can be obtained by elementary row and column operations on the parity-check matrix. Finally, we show that the free distance of the convolutional codes is equal to the minimum distance of their block counterparts.

1. Introduction

Low-density parity-check (LDPC) codes are a class of linear block codes represented by sparse parity-check matrices [1], capable of performing very close to the Shannon capacity limits when they are decoded under simple iterative decoders [2], [3], such as sum-product algorithm [4].

Cycle codes [5] are a special class of binary LDPC codes with the property that each column of the parity-check matrix contains exactly two nonzero elements. Gallager [1] has shown that the minimum distance of cycle codes increases logarithmically with the code length, while this increment is linear for the codes with column-weight greater than two. In spite of this weakness which causes poor performance, cycle codes have their own advantages: 1) Their encoding and decoding operations have lower computation and storage complexity, 2) They have better block error statistics when applied on partial response channels [9]-[11], 3) Non-binary cycle codes are

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among the most promising non-binary codes and for q = 64, 128, 256, the best q-array LDPC codes decoded with belief propagation (BP) algorithm are cycle codes [12]-[13]. This makes non-binary cycle codes good candidates for both optimum maximum likelihood (ML) and iterative decoding, 4) Compared with other LDPC codes, the girth of the Tanner graph plays more important roles for cycle codes [14, 15], since it affects not only the message dependence of iterative decoding but also the minimum distance of the code [13].

Convolutional codes [20] are a type of error-correcting codes which are used extensively in numerous applications in order to achieve reliable data transfer, including digital video, radio, mobile communication, and satellite communication. The free distance [18] of a convolutional code determines to a large extent the error rate in the case of maximum likelihood (ML) decoding, when applied on some decoding algorithms, such as Viterbi decoder [19] with finite path register length.

Some authors [6],[7],[8] have investigated the relations between QC LDPC codes and convolutional codes. Following the approach introduced by [7], we have constructed convolutional codes associated to Type I, II, III LDPC codes [14], [15], [17] with maximum-girths 16, 24 and 12, respectively, where the generator matrix $G(D)$ can be obtained by elementary row and column operations on the parity-check matrix $H(D)$. Also, it is proved that the free distance d_{free} of such convolutional codes is equal to the minimum distance of their corresponding QC block codes. By extending Type I-III LDPC codes to cylinder type block-circulant (CTBC) cycle codes, with desired maximum-girth, as an arbitrary multiple of 8 [16], here we obtain the general form of the generator matrix $G(D)$ associated with the convolutional CTBC cycle codes, and then we prove that the free distance of such convolutional codes is equal to the minimum distance of their QC counterparts.

2. **CTBC Cycle Codes**

Let m, s be nonnegative integers with $0 \leq s \leq m-1$. The $m \times m$ circulant permutation matrix shifted by s, I_m^s, is the matrix obtained from $m \times m$ identity matrix I_m by shifting each row s positions to the bottom, that is $I_m^s = (e_{i,j})_{m\times m}$ where $e_{i,j} = 1$ if $i - j = s \bmod m$ and $e_{i,j} = 0$, elsewhere. It is clear that $I_m^0 = I$. For simplicity, I_m^s can be denoted by I^s when m is known.

An low-density parity-check (LDPC) code is called block-circulant, if, for some positive integer $m > 1$, the parity-check matrix consists of $m \times m$ circulant permutation matrices and $m \times m$ zero matrix. A particular class of quasi-cyclic block-circulant (BC) LDPC codes, referred to as cylinder type BC (CTBC) LDPC codes has been introduced in [16]. Also, it is shown that the maximum-girth of a class of column-weight two CTBC codes, or CTBC cycle codes, denoted by the parity check matrices $H_{m,e,p,s}$, is $8(1+e)$, where e is an arbitrary positive integer. On the other word, $g(H_{m,e,p,s}) \leq 8(e+1)$. Moreover, for each e, there are some positive integers m, p and slop-vectors s, such that the equality is hold, i.e. $g(H_{m,e,p,s}) = 8(e+1)$. In fact $H_{m,e,p,s} = (h_{i,j})_{1\leq i\leq pe, 1\leq j\leq p(e+1)}$, where:

$$h_{i,j} = \begin{cases} I & \begin{array}{l} i = ek_1, j \in \{(e+1)k_1 - 1, (e+1)k_1\} \text{ or} \\ i = ek_2 + 1, j = (e+1)k_2 - 1 \text{ or} \\ j = (e+1)k_3 + t, i \in \{ek_3 + t, ek_3 + t + 1\}, 1 \leq t \leq e; \end{array} \\ I^{s_{k_4}} & i = ek_4 + 1, j = (e+1)k_4; \\ I^{s_{p+1-t}} & i = 1, j = (e+1)p - t, 0 \leq t \leq 1; \\ 0 & \text{otherwise}; \end{cases} \tag{2.1}$$

and $0 \leq s_i \leq m-1$ for each $1 \leq i \leq p+1$. The vector $s = (s_1, \cdots, s_{p+1})$ is called the slop-vector of $H_{m,e,p,s}$. CTBC cycle codes with the the parity-check matrices $H_{m,1,p,s}$ and $H_{m,2,p,s}$ are referred to as Type I and Type II codes with maximum-girths 16 and 24, respectively [14]. For example:

$$H_{m.1,3,s} = \begin{pmatrix} I & I & & & I^{s_3} & I^{s_4} \\ I & I^{s_1} & I & I & & \\ & & I & I^{s_2} & I & I \end{pmatrix}, \quad H_{m.2,2,s} = \begin{pmatrix} I & & & & I^{s_2} & I^{s_3} \\ I & I & I & & & \\ & I & I^{s_1} & I & & \\ & & & I & I & I \end{pmatrix}$$

The rate of the CTBC cycle code with the parity-check matrix $H_{m,e,p,s}$ is $\frac{1}{e+1}$, which tends to zero by increment of e. Because of the relation $d = g/2$ for the cycle codes with girth g and minimum distance d, the minimum distance of the code can achieve the maximum value $4(1+e)$.

3. **Convolutional CTBC Codes**

Following the approach used to get convolutional codes from quasi-cyclic LDPC codes [7], [16], the parity-check matrix $H(D)$ of a convolutional CTBC cycle code can be derived by substituting each permutation circulant I^s in H by D^s and I by 1, where D stands for the delay operation. It is noticed that the parity-check matrix $H(D)$ of the corresponding LDPC convolutional code, is over the field $F_2(D)$, while the polynomial form of the associated parity-check matrix of QC CTBC codes is over the ring $F_2[D]/(D^m+1)$. Now, let $H(D) = (h_{i,j}(D))_{1 \leq i \leq pe, 1 \leq j \leq p(e+1)}$ be the parity-check

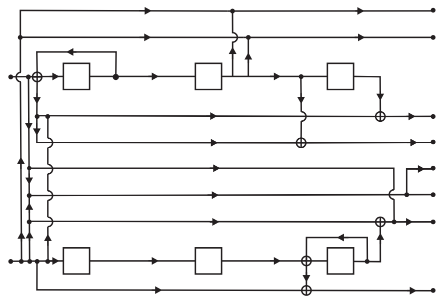

FIGURE 1. (2,8) convolution code represented by example 3.1.

matrix of the convolutional CTBC cycle code with the parity-check matrix $H_{m,e,p,s}$, where:

$$h_{i,j}(D)=\begin{cases} 1 & i=ek_1, j=(e+1)k_1 \text{ or} \\ & j=(e+1)k_3+t, i\in\{ek_3+t, ek_3+t+1\}, 1\le t\le e; \\ D^{s_{k_4}} & i=ek_4+1, j=(e+1)k_4; \\ D^{s_{p+1-t}} & i=1, j=(e+1)p-t, 0\le t\le 1; \\ 0 & \text{otherwise;} \end{cases}$$

By elementary row and column operations on the parity-check matrix $H(D)$, we can find the generator matrix $G(D)$.

Example 3.1. Let $H(D)$ be the convolutional form of $H_{m,3,3,s}$, represented by:

$$H(D)=\begin{pmatrix} 1 & & & & & & D^{s_2} & D^{s_3} \\ 1 & 1 & & & & & & \\ & 1 & 1 & 1 & & & & \\ & & 1 & D^{s_1} & 1 & & & \\ & & & & 1 & 1 & & \\ & & & & & 1 & 1 & 1 \end{pmatrix}$$

Applying the elementary row and column operations, we obtain the following generator matrix $G(D)$:

$$G(D)=\begin{pmatrix} D^{s_2} & D^{s_2} & \frac{1+D^{s_1+s_2}}{1+D^{s_1}} & \frac{1+D^{s_2}}{1+D^{s_1}} & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & \frac{1+D^{s_3}}{D^{s_2}+D^{s_3}} & \frac{1+D^{s_2}}{D^{s_2}+D^{s_3}} \end{pmatrix}$$

Encoder of this (2,8)-convolution code is drawn in Figure 1.

Example 3.2. Let $H(D)$ be the convolutional form of $H_{m,3,3,s}$, represented by:

$$H(D)=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&D^{s_3}&D^{s_4}\\ 1&1&0&0&0&0&0&0&0&0&0&0\\ 0&1&1&1&0&0&0&0&0&0&0&0\\ 0&0&1&D^{s_1}&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&1&1&0&0&0&0\\ 0&0&0&0&0&0&1&D^{s_2}&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&1 \end{pmatrix}$$

Applying the elementary row and column operations, we obtain the following generator matrix $G(D)$:

$$G(D)=\begin{pmatrix} D^{s_3}&D^{s_3}&\frac{1+D^{s_1+s_3}}{1+D^{s_1}}&\frac{1+D^{s_3}}{1+D^{s_1}}&1&1&1&0&1&1&1&0\\ D^{s_3}&D^{s_3}&D^{s_3}&0&D^{s_3}&D^{s_3}&\frac{1+D^{s_2+s_3}}{1+D^{s_2}}&\frac{1+D^{s_3}}{1+D^{s_2}}&1&1&1&0\\ 1&1&1&0&1&1&1&0&1&1&\frac{1+D^{s_4}}{D^{s_3}+D^{s_4}}&\frac{1+D^{s_3}}{D^{s_3}+D^{s_4}} \end{pmatrix}$$

Deriving the generator matrix $G(D)$ of a convolutional LDPC code, from its parity-check matrix $H(D)$ is not easy in general. But, for CTBC cycle codes with the parity-check matrix $H_{m,e,p,s}$, the following lemma gives us the general form of the generator matrix $G(D)$.

Theorem 3.3. $G(D)=(g_{i,j}(D))_{1\le i\le p,1\le j\le p(e+1)}$ is the generator matrix of the convolutional CTBC cycle code with the parity-check matrix $H_{m,e,p,s}$, where:

$$g_{i,j}(D)=\begin{cases} 1, & \begin{array}{l}1\le i\le p-1, j\ge i(e+1)+1, e+1\nmid j, \text{ or}\\ i=p, 1\le j\le p(e+1)-2, e+1\nmid j\end{array}\\ D^{s_p}, & 1\le i\le p-1, j\le i(e+1)-2, e+1\nmid j,\\ \frac{1+D^{s_i+s_p}}{1+D^{s_i}}, & 1\le i\le p-1, j=i(e+1)-1,\\ \frac{1+D^{s_p}}{1+D^{s_i}}, & 1\le i\le p-1, j=i(e+1),\\ \frac{1+D^{s_p+1}}{D^{s_p}+D^{s_p+1}}, & (i,j)=(p,p(e+1)-1),\\ \frac{1+D^{s_p}}{D^{s_p}+D^{s_p+1}}, & (i,j)=(p,p(e+1)),\\ 0, & \text{otherwise;} \end{cases}$$

In other words, the $p\times p(e+1)$ matrix $G(D)$ is in the following form:

$$G(D)=\begin{pmatrix} A & \cdots & A & B_1 & C_1 & 1 & \cdots & 1 & 1 & 0 & 1 & \cdots & 1 & 0 & 1 & \cdots & 1 & 1 & 0\\ A & \cdots & A & A & 0 & A & \cdots & A & B_2 & C_2 & 1 & \cdots & 1 & 0 & 1 & \cdots & 1 & 1 & 0\\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ A & \cdots & A & A & 0 & A & \cdots & A & A & 0 & A & \cdots & B_{p-1} & C_{p-1} & 1 & \cdots & 1 & 1 & 0\\ 1 & \cdots & 1 & 1 & 0 & 1 & \cdots & 1 & 1 & 0 & 1 & \cdots & 1 & 0 & 1 & \cdots & 1 & E & F \end{pmatrix}$$

wherein $A=D^{s_p}$, $B_i=\frac{1+D^{s_i+s_p}}{1+D^{s_i}}$, $C_i=\frac{1+D^{s_p}}{1+D^{s_i}}$, for each $1\leq i\leq p-1$, $E=\frac{1+D^{s_{p+1}}}{D^{s_p}+D^{s_{p+1}}}$ and $F=\frac{1+D^{s_p}}{D^{s_p}+D^{s_{p+1}}}$.

Proof 3.4. *Because of the existence of p independent columns in $G(D)$, i.e. the columns containing $C_1,\cdots,C_{p-1}$, F, the rank of $G(D)$ is p. So, it is sufficient to prove that $G(D)H(D)^T=0$, or equivalently each row of $G(D)$ and each row of $H(D)$ are orthogonal. On the other word, we must prove that:*

$$\sum_{j=1}^{p(e+1)} g_{i,j}(D)h_{i',j}(D)=0 \tag{3.1}$$

for each $1\leq i\leq pe$ and $1\leq i'\leq p$. So, we consider two following cases:

(1) *If $2\leq i'\leq p$, then by the Euclidean algorithm, there are unique integers k, $k<p$, and t, $1\leq t\leq e$, such that $i'=ek+t$. Now if $t=e$, then the left-hand of the equation 3.1 is equal to:*

$$g_{i,(e+1)k+e-1}(D)+g_{i,(e+1)k+e}(D)+g_{i,(e+1)k+e+1}(D). \tag{3.2}$$

If $t=1$, then the left-hand of the equation 3.1 is equal to:

$$g_{i,(e+1)k+1}(D)+D^{s_k}g_{i,(e+1)k+2}(D)+g_{i,(e+1)k+3}(D). \tag{3.3}$$

Also, if $1<t<e$, then the left-hand of the equation 3.1 is equal to:

$$g_{i,(e+1)k+t-1}(D)+g_{i,(e+1)k+t}(D). \tag{3.4}$$

Now, we prove that the equations 3.2, 3.3 and 3.4 are equal to zero in the ring $F_2(D)$. For simplicity, we just prove that the equation 3.2 is zero. The others are similar. We have consider two following cases:

(a) *If $i<p$. Then we have three following cases:*

(i) *If $k\geq i$, then $(e+1)k+e\geq(e+1)k+e-1\geq i(e+1)+1$, so $g_{i,(e+1)k+e-1}(D)=g_{i,(e+1)k+e}(D)=1$; but $g_{i,(e+1)k+e+1}(D)=0$, because $e+1\mid(e+1)k+e+1$, but $k+1>i$, and $(e+1)k+e+1\neq i(e+1)$. So the equation 3.2 is equal to zero.*

(ii) *If* $k = i - 1$*, then* $(e+1)k + e + 1 = i(e+1)$*. So* $g_{i,(e+1)k+e+1}(D) = \frac{1+D^{s_p}}{1+D^{s_i}}$*. Also* $(e+1)k + e = i(e+1) - 1$*, then we have* $g_{i,(e+1)k+e}(D) = \frac{1+D^{s_i+s_p}}{1+D^{s_i}}$*. But* $(e+1)k + e - 1 \leq i(e+1) - 2$*. Therefore* $g_{i,(e+1)k+e-1}(D) = D^{s_p}$*, and the equation 3.2 is equal to* $g_{i,(e+1)k+e-1}(D) = D^{s_p} + \frac{1+D^{s_i+s_p}}{1+D^{s_i}} + \frac{1+D^{s_p}}{1+D^{s_i}} = 0$.

(iii) *If* $k < i-1$*, then* $(e+1)k+e-1 \leq (e+1)k+e \leq i(e+1)-2$*. So* $g_{i,(e+1)k+e-1}(D) = g_{i,(e+1)k+e}(D) = D^{s_p}$*. But* $e+1 \mid (e+1)k+e+1$*, and* $(e+1)k+e+1 \neq i(e+1)$*. Then* $g_{i,(e+1)k+e+1}(D) = 0$*. So the equation 3.2 is equal to* $D^{s_p} + D^{s_p} + 0 = 0$.

(b) *If* $i = p$*. Then we have two following cases;*

(i) *If* $k < p-1$*, then* $(e+1)k+e-1 \leq (e+1)k+e \leq p(e+1)-2$*. So* $g_{i,(e+1)k+e-1}(D) = g_{i,(e+1)k+e}(D) = 1$*. But* $e+1 \mid (e+1)k+e+1$*, and* $i = p$*. Then* $g_{i,(e+1)k+e+1}(D) = 0$*. So the equation 3.2 is equal to* $1 + 1 + 0 = 0$.

(ii) *If* $k = p - 1$*, then* $(e+1)k + e + 1 = p(e+1)$*. So* $g_{i,(e+1)k+e+1}(D) = \frac{1+D^{s_p}}{D^{s_p}+D^{s_{p+1}}}$*. Also* $(e+1)k + e = p(e+1) - 1$*, then we have* $g_{i,(e+1)k+e}(D) = \frac{1+D^{s_{p+1}}}{D^{s_p}+D^{s_{p+1}}}$*. But* $(e+1)k + e - 1 = p(e+1) - 2$*, while* $e+1 \nmid p(e+1) - 2$*. So* $g_{i,(e+1)k+e-1}(D) = 1$*, and the equation 3.2 is equal to* $1 + \frac{1+D^{s_{p+1}}}{D^{s_p}+D^{s_{p+1}}} + \frac{1+D^{s_p}}{D^{s_p}+D^{s_{p+1}}} = 0$.

(2) *If* $i' = 1$*, then the left-hand of the equation 3.1 is equal to:*

$$g_{i,1}(D) + D^{s_p} g_{i,(e+1)p-1}(D) + D^{s_{p+1}} g_{i,(e+1)p}(D). \tag{3.5}$$

We consider two following cases:

(a) *If* $1 \leq i \leq p-1$*, then* $g_{i,(e+1)p}(D) = 0$ *and* $g_{i,(e+1)p-1}(D) = 1$*. But* $g_{i,1}(D) = D^{s_p}$*. So, the equation 3.5 is equal to* $D^{s_p} + D^{s_p} + 0 = 0$

(b) *If* $i = p$*, then* $g_{i,1}(D) = 1$*,* $g_{i,(e+1)p-1}(D) = \frac{1+D^{s_{p+1}}}{D^{s_p}+D^{s_{p+1}}}$ *and* $g_{i,(e+1)p}(D) = \frac{1+D^{s_p}}{D^{s_p}+D^{s_{p+1}}}$*. So the equation 3.5 is equal to* $1 + D^{s_p}(\frac{1+D^{s_{p+1}}}{D^{s_p}+D^{s_{p+1}}}) + D^{s_{p+1}}(\frac{1+D^{s_p}}{D^{s_p}+D^{s_{p+1}}}) = 0$.

4. Free Distance of Convolutional CTBC Cycle Codes

Now, we prove that the minimum-distance of each CTBC cycle code with maximum girth is equal to the free distance of its convolutional form.

Theorem 4.1. Let $H_{m,e,p,s}$ be the parity-check matrix of a CTBC cycle code with maximum-girth $8(e+1)$ and minimum distance d. Also, let $H(D)$ be the parity-check matrix of its corresponding convolutional code with free distance d_{free}. Then we have $d = d_{\text{free}}$.

Proof 4.2. *It is verified that the minimum distance of a cycle code* C *is equal to half girth; so we have* $d = 4(1+e)$*. If* d_{free} *is the free distance of the convolutional form of* C*, then it is proved that* $d \leq d_{\text{free}}$ *[8]. Therefore, it suffices to show that* $d_{\text{free}} \leq 4(1+e)$*. The vector*

$u(D)=(\frac{1+D^{s_p}}{1+D^{s_1}},\frac{1+D^{s_p}}{1+D^{s_2}},0,\cdots,0)$ *is encoded in the length-*$p(e+1)$ *vector* $v(D)=u(D)G(D)$ *by:*

$$v(D)=(v_0(D),\cdots,v_{p(e+1)-1}(D))=u(D)G(D)=\frac{(1+D^{s_1})(1+D^{s_2})}{1+D^{s_p}}\times$$

$$\begin{array}{l}[(D^{s_p}\ \cdots\ D^{s_p}\ \ \frac{1+D^{s_1+s_p}}{1+D^{s_1}}\ \ \frac{1+D^{s_p}}{1+D^{s_1}}\ \ 1\ \cdots\ 1\ \ 1\ \ 0\ \ 1\ \cdots\ 1\ \ 0)\ +\\ (D^{s_p}\ \cdots\ D^{s_p}\ \ D^{s_p}\ \ 0\ \ D^{s_p}\ \cdots\ D^{s_p}\ \ \frac{1+D^{s_2+s_p}}{1+D^{s_2}}\ \ \frac{1+D^{s_p}}{1+D^{s_2}}\ \ 1\ \cdots\ 1\ \ 0)]\ =\\ (0\ \cdots\ 0\ \ J(D)\ \ J(D)\ \ K(D)\ \cdots\ K(D)\ \ L(D)\ \ M(D)\ \ 0\ \cdots\ 0\ \ 0).\end{array}$$

where

$J(D)=1+D^{s_2}$, $K(D)=(1+D^{s_1})(1+D^{s_2})$, $L(D)=D^{s_2}(1+D^{s_1})$ *and* $M(D)=1+D^{s_1}$.

But, each of the elements 0 *and* $K(D)$ *is repeated* $e-1$ *times in* $v(D)$*. So, the codeword associated to* $v(D)$ *is:*

$$v_0(D^{p(e+1)})+Dv_0(D^{p(e+1)})+\cdots+D^{p(e+1)-1}v_{p(e+1)-1}(D^{p(e+1)})=$$

$$D^{e-1}J(D^{p(e+1)})+D^{e}J(D^{p(e+1)})+K(D^{p(e+1)})\sum_{i=e+1}^{2e-1}D^i+D^{2e}L(D^{p(e+1)})+D^{2e+1}M(D^{p(e+1)}),$$

which has the weight at most $2+2+4(e-1)+2+2=4(e+1)$*; because, the weight of* $J(D)$*,* $L(D)$ *and* $M(D)$ *is 2 and the weight of* $K(D)$ *is 4. So* $d_{\text{free}}\le 4(e+1)$*. On the other hand,* $d_{\text{free}}\ge d=4(e+1)$*. Then we have* $d=d_{\text{free}}=4(e+1)$*.*

5. Conclusion

A class of column-weight two quasi-cyclic LDPC codes, called cylinder-type block-circulant (CTBC) cycle codes, was applied to generate a class of LDPC convolutional codes. The general form of generator matrices $G(D)$ of the constructed convolutional CTBC cycle codes was formulated. It was shown that the minimum distance of the considered CTBC cycle codes with maximum-girth is equal to the free distance of the associated convolutional codes.

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