https://toc.ui.ac.ir

Transactions on Combinatorics
ISSN (print): 2251-8657, ISSN (on-line): 2251-8665
Vol. x No. x (202x), pp. xx-xx.

www.ui.ac.ir

MATCHING INTEGRAL GRAPHS WITH MAXIMUM VERTEX DEGREE FOUR

SOMAYEH KHALASHI GHEZELAHMAD

Abstract. The matching polynomial of a graph has coefficients that give the number of matchings in the graph. In this paper, we determine all connected graphs with maximum vertex degree four whose matching polynomials have only integer zeros. A graph is matching integral if the zeros of its matching polynomial are all integers. We show that there are exactly two matching integral graphs with maximum vertex degree four.

1. **Introduction**

Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. By *order* and *size* of G, we mean the number of vertices and the number of edges of G, respectively and we denote it by $|V(G)|$ and $|E(G)|$, respectively. The maximum degree of G is $\Delta(G)$ (or Δ if G is clear from the context). We denote the complete graph of order n by K_n and the star graph of order $n+1$ by $K_{1,n}$. For any vertex u of G, we use $deg(u)$ to denote the degree of u. An *r-matching* in G is a set of r pairwise non-incident edges. The number of r-matchings in G is denoted by $p(G, r)$. It is consistent to define $p(G, 0) = 1$. The *matching polynomial* of G is given by

$$\mu(G, x) = \sum_{r=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^r p(G, r) x^{n-2r},$$

MSC(2010): Primary: 05C31; Secondary: 05C70, 05C50.

Keywords: Matching integral, Matching polynomial, Matching zero.

Article Type: Research Paper.

Communicated by Hamidreza aimani.

Received: 22 November 2024, Accepted: 22 September 2025, Published Online: 12 November 2025.

Cite this article: S. Khalashi Ghezelahmad, Matching integral graphs with maximum vertex degree four, *Trans. Comb.*, **x** no. x (202x) xx-xx. https://dx.doi.org/10.22108/toc.2025.143470.2225 .

where $n = |V(G)|$. By way of example, the matching polynomial of a path on four vertices is $x^4 - 3x^2 + 1$. The roots of $\mu(G,x)$ are called the *matching roots* (or *matching zeros*) of G. The notation $\mathrm{mult}(\theta, G)$ is used for the multiplicity of θ as a zero of $\mu(G,x)$. We also denote the multiset of the roots of the matching polynomial of G by $R(G)$. We use exponent symbol to show the multiplicities of the elements of $R(G)$.

The matching polynomial is related to the characteristic polynomial of G, which is defined to be the characteristic polynomial of the adjacency matrix of G. In particular, these two coincide if and only if G is a forest [7]. Also the matching polynomial of any connected graph is a factor of the characteristic polynomial of some tree [6]. The theory of mathcing polynomial is well elaborated in [3, 4, 6, 7, 8, 9].

A graph is said to be *integral* if eigenvalues of its adjacency matrix consist entirely of integers. The notion of integral graphs dates back to Harary and Schwenk [10]. Furthermore, several explicit constructions of integral graphs of special types appear in the literature, see [2] and references therein.

The idea of studying graphs with integer matching polynomial zeros was first appeared in [1], where Akbari, Csikvari, etc., introduced the concept of matching integral graphs. A graph is called *matching integral* if all zeros of its matching polynomial are integers. They characterized all traceable graphs which are matching integral. They studied matching integral regular graphs and showed that for $k \geq 2$ there is only one connected matching integral k-regular graph, namely $K_7 \backslash (E(C_3) \cup E(C_4))$. Furthermore, it has been proved that there is no matching integral claw-free graph and K_2 is the only connected matching integral graph with a perfect matching.

Matching integral graphs of small order have been studied by the author. It has been shown that there are exactly ten connected matching integral graphs up to eight vertices and there are four connected matching integral graphs with maximum vertex degree at most three, see [12, 13]. Motivated by the previous works, we are interested to characterize matching integral graphs with maximum vertex degree four. In this paper, we determine such graphs and we show that there are only two connected graphs with maximum vertex degree four whose all matching zeros are integers. Our work has been organized into two sections. Section 2, contains necessary background information and various preliminary-type results including matching polynomial of a graph. Section 3, is devoted to studying matching integral graphs with maximum vertex degree four.

2. Preliminaries

In order to establish our results, we need the following theorems and lemmas:

Remark 2.1. *A graph with odd order has* 0 *as a matching root and if* θ *is a matching zero of a graph, then so is* $-\theta$*.*

https://dx.doi.org/10.22108/toc.2025.143470.2225

Remark 2.2. *Let n and m be the order and size of a graph G, respectively. Let $\theta_1, \ldots, \theta_k$ be the positive zeros of $\mu(G,x)$, not necessarily distinct. Then $m = p(G,1) = \sum_{i=1}^{k} \theta_i^2$ and $p(G,2) = \sum_{i \neq j} \theta_i^2 \theta_j^2$.*

Theorem 2.3. [11] *For any graph G, the zeros of $\mu(G,x)$ are all real. If $\Delta > 1$, then the zeros lie in the interval $(-2\sqrt{\Delta - 1},\ 2\sqrt{\Delta - 1})$.*

Theorem 2.4. [5] *If G is a connected graph, then the largest zero of $\mu(G,x)$ has multiplicity 1. In other words, it is a simple zero.*

Let $t(G)$ be the number of vertices of a longest path in the graph G.

Theorem 2.5. [11] *Let G be a graph. Then the following holds:*

(a) *The maximum multiplicity of a zero of $\mu(G,x)$ is at most equal to the number of vertex-disjoint paths required to cover G.*
(b) *The number of distinct zeros of $\mu(G,x)$ is at least $t(G)$.*
(c) *In particular, if the graph G is traceable then all zeros of $\mu(G,x)$ are simple.*

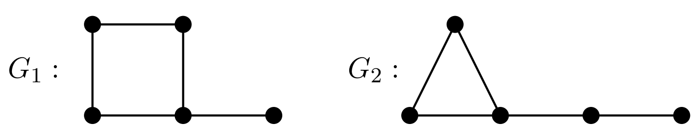
Theorem 2.6. [6] *If $\theta = 0$, then $\mathrm{mult}(\theta, G)$ is the number of vertices in G missed by a maximum matching.*

Theorem 2.7. [11] *Let G be a graph and u be a vertex of it. Then the zeros of $\mu(G \setminus u, x)$ interlace those of $\mu(G,x)$, i.e if $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$ and $\eta_1 \geq \eta_2 \geq \cdots \geq \eta_{n-1}$ are matching zeros of G and $G \setminus u$, respectively, then*

$$\theta_1 \geq \eta_1 \geq \theta_2 \geq \eta_2 \geq \cdots \geq \eta_{n-1} \geq \theta_n.$$

Theorem 2.8. [14] *For a connected graph G, if $\mathrm{mult}(\theta, G) \geq 2$ then there is a vertex u of G such that $\mathrm{mult}(\theta, G \setminus \mathrm{u}) = \mathrm{mult}(\theta, G) + 1$.*

Theorem 2.9. [1] *Let G be a connected graph whose matching polynomial has only simple integer zeros. Then G is one of the following graphs: K_1, K_2, $K_7 \setminus (E(C_3) \cup E(C_4))$, G_1 or G_2, where*

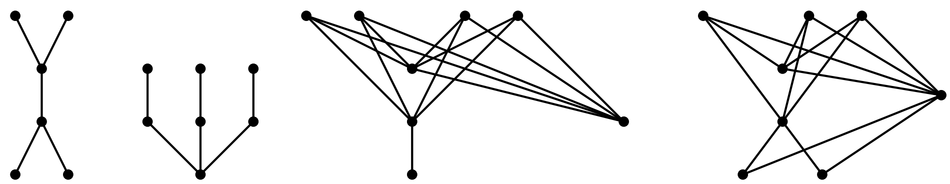


In particular, this is the list of matching integral traceable graphs.

The following theorem, describes all connected matching integral graphs of order at most eight.

Theorem 2.10. [12, 13] *Let G be a connected matching integral graph whose order is at most eight. Then G is either $K_{1,4}$, or one of the graphs listed in Theorem 2.9 or one of the following graphs:*

https://dx.doi.org/10.22108/toc.2025.143470.2225



Theorem 2.11. [1] *If a graph G has a perfect matching, then its matching polynomial has a zero in the interval $(0, 1]$. If it has no zero in the interval $(0, 1)$ then it is the disjoint union of some K_2.*

It follows from Theorem 2.11 that, the only connected matching integral graph which has a perfect matching is K_2.

By following Ghorbani's definition [4], $\mathcal{G}(r, k, t; p, q)$ is the set of graphs obtained by adding a new vertex u to the graph $rK_{1,k} \cup tK_1$ and joining it to the other vertices by p edges such that the resulting graph is connected and u is adjacent with exactly q centers of the stars. Clearly, $r+t \leq p \leq r(k+1)+t$ and $0 \leq q \leq r$. The notations $K(k, t; l)$ and $K'(k, t; l)$ are used for the graphs $\mathcal{G}(1, k, t; l + t, 0)$ and $\mathcal{G}(1, k, t; l + t + 1, 1)$, respectively. Their matching polynomials are

$$\mu(K(k, t; l), x) = x^{k+t-2}(x^4 - (k + t + l)x^2 + (l + t)(k - 1) + t), \tag{2.1}$$

$$\mu(K'(k, t; l), x) = x^{k+t-2}(x^4 - (k + t + l + 1)x^2 + (l + t)(k - 1) + t). \tag{2.2}$$

It is easily seen that $\Delta(K(k, t; l)) = \mathrm{Max}\{t + l, k\}$ and $\Delta(K'(k, t; l)) = \mathrm{Max}\{t + l + 1, k + 1\}$.

The graph $\mathcal{G}(r, 1, 0; s, q)$, where $q = s - r$ and $r \leq s \leq 2r$ is denoted by $S(r, s)$. Its matching polynomial is

$$\mu(S(r, s), x) = x(x^2 - s - 1)(x^2 - 1)^{r-1}. \tag{2.3}$$

The graph $\mathcal{G}(r, k, 0; r, r)$ is denoted by $T(r, k)$ and its matching polynomial is

$$\mu(T(r, k), x) = x^{r(k-1)+1}(x^2 - r - k)(x^2 - k)^{r-1}. \tag{2.4}$$

It is easy to see that $\Delta(S(r, s)) = s$ and $\Delta(T(r, k)) = \mathrm{Max}\{r, k + 1\}$.

For any $G \in \mathcal{G}(r, 3, t; p, q)$ the set of graphs obtained by adding s copies of K_3 to G and joining them by l edges to the vertex u of G to make a connected graph is denoted by $\mathcal{H}(r, s, t; p, q, l)$. The notation $L(t, l)$ is used for the graph $\mathcal{H}(0, 1, t; t, 0, l)$, for $l = 1, 2, 3$ and its matching polynomial is

$$\mu(L(t, l), x) = x^t(x^4 - (t + l + 3)x^2 + 3t + l). \tag{2.5}$$

We have $\Delta(L(t, l)) = \mathrm{Max}\{t + l, 3\}$.

https://dx.doi.org/10.22108/toc.2025.143470.2225

Theorem 2.12. [4] *Let G be a connected graph and $z(G)$ be the number of its distinct matching zeros.*

(i) *If $z(G) = 2$, then $G = K_2$.*

(ii) *If $z(G) = 3$, then G is either a star or K_3.*

(iii) *If $z(G) = 4$, then G is a non-star graph with 4 vertices.*

(iv) *If $z(G) = 5$, then G is one of the graphs $K(k,t;l)$, $K'(k,t;l)$, $L(t,l)$, $T(r,k)$, $S(r,s)$, for some integers k, t, l, r, s or a connected non-star graph with 5 vertices.*

3. Matching Integral Graphs with Maximum Vertex Degree Four

In this section, we characterize all connected matching integral graphs with maximum vertex degree four. In addition, we determine all matching integral graphs whose longest path avoids exactly one vertex.
From (2.3) and (2.4), it follows that:

Lemma 3.1. *The Graph $S(r,s)$ is matching integral if and only if $s+1$ is square.*

Lemma 3.2. *The Graph $T(r,k)$ is matching integral if and only if k and $r+k$ are square.*

Theorem 3.3. *The only connected matching integral graphs with $\Delta = 4$ are $K_{1,4}$ and $K_7 \backslash (E(C_3) \cup E(C_4))$.*

Proof. Let n and m be the order and size of G, respectively. Since $\Delta = 4$ and G is matching integral, by Theorem 2.3 matching zeros of G, lie in the interval $[-3,3]$. Thus $z(G) \leq 7$. Note that, if $n \leq 8$, then by Theorem 2.10, we find that G is either $K_{1,4}$ or $K_7 \setminus (E(C_3) \cup E(C_4))$. Hence we may assume that $n \geq 9$. Now, from Theorem 2.12, we deduce that $z(G) \geq 5$. Furthermore, Theorems 2.11 and 2.6, imply that G does not have a perfect matching and so zero is a matching root of G. This excludes the possibility of $z(G) = 6$. We consider two cases:

Case 1. $z(G) = 5$. Hence in view of Theorem 2.12 (iv), one of the following cases may occur:

(a) G is one of the graphs $K(k,t;l)$, $K'(k,t;l)$ or $L(t,l)$ for some integers k,t,l. First note that since $n \geq 9$, $m \geq 8$. Furthermore, from (2.1), (2.2), (2.5) we see that the root system of G is of the form $R(G) = \{0^r, \pm\alpha, \pm\beta\}$, $r \geq 2$, $0 < \alpha < \beta \leq 3$. Thus $m = \alpha^2 + \beta^2 \in \{10, 13\}$. Now, if G is a $K(k,t;l)$, then since $m = k+t+l$ and $\Delta = \text{Max}\{t+l, k\} = 4$, we get $m \leq 8$, a contradiction. If G is a $K'(k,t;l)$, then since $m = k+t+l+1$ and $\Delta = \text{Max}\{t+l+1, k+1\} = 4$, we have $m \leq 7$, a contradiction. If G is an $L(t,l)$, then from $m = t+l+3$ and $\Delta = \text{Max}\{t+l, 3\} = 4$, we obtain $m = 7$, a contradiction.

(b) G is a $T(r,k)$ for some integers r, k. Now, by Lemma 3.2 we know that G is matching integral if and only if k and $r+k$ are square. However, it is easy to see that since $\Delta = \text{Max}\{r, k+1\} = 4$, this is not possible.

(c) G is an $S(r,s)$ for some integers r, s. Since $\Delta = s = 4$, by Lemma 3.1, we see that G is not matching integral.

Case 2. $z(G) = 7$. Thus Theorem 2.4 yields that $R(G) = \{0^t, (\pm 1)^r, (\pm 2)^s, \pm 3\}$ where t, r, s are positive integers. We have $n = t + 2(r+s+1)$ and $m = r + 4s + 9$. Let a_i be the number of vertices with degree i. Then $a_i = 0$ if $i > \Delta$ or $i = 0$ (G is connected), and $\sum a_i = n$ and $\sum i a_i = 2m$. Moreover, $p(G,2) = \binom{m}{2} - \sum \binom{deg(u)}{2}$, which implies that $\sum i^2 a_i = \sum deg(u)^2 = m(m+1) - 2p(G,2)$. It is seen that one of the following cases may occur:

(a) $r = s = 1$. Hence $n = t+6$, $m = 14$ and $p(G,2) = 49$. We have

$$\sum_{i=1}^{4} a_i = t+6, \qquad \sum_{i=1}^{4} i a_i = 28, \qquad \sum_{i=1}^{4} i^2 a_i = 112.$$

Then

$$\begin{aligned} 0 \geq -2(a_2 + a_3) &= \sum_{i=1}^{4} (i-1)(i-4) a_i \\ &= \left(\sum_{i=1}^{4} i^2 a_i\right) - 5\left(\sum_{i=1}^{4} i a_i\right) + 4\left(\sum_{i=1}^{4} a_i\right) \\ &= 4t - 4, \end{aligned}$$

so $t = 1$ and $n = 7$, a contradiction.

(b) $r \geq 2$, $s = 1$. Thus $n = t + 2r + 4$, $m = r + 13$ and $p(G,2) = \binom{r}{2} + 13r + 36$. Therefore

$$\sum_{i=1}^{4} a_i = t + 2r + 4, \qquad \sum_{i=1}^{4} i a_i = 2r + 26, \qquad \sum_{i=1}^{4} i^2 a_i = 2r + 110.$$

Then

$$\begin{aligned} 0 &\geq \sum_{i=1}^{4} (i-1)(i-4) a_i \\ &= \left(\sum_{i=1}^{4} i^2 a_i\right) - 5\left(\sum_{i=1}^{4} i a_i\right) + 4\left(\sum_{i=1}^{4} a_i\right) \\ &= 4t - 4, \end{aligned}$$

and so $t = 1$. Now, by Theorem 2.8 there is a vertex u of G such that $R(G-u) = \{(\pm 1)^{r+1}, \pm\eta\}$, for some η, which by interlacing, $2 \leq \eta < 3$. Let m' be the size of $G - u$. Thus $m' = \eta^2 + r + 1$. Moreover, since $G - u$ has more that four vertices, Theorem 2.12 (iii) implies that $G - u$ is not connected. It follows that $G - u = rK_2 \cup G_1$, where G_1 is a connected graph on four vertices with

https://dx.doi.org/10.22108/toc.2025.143470.2225

$R(G_1) = \{\pm 1, \pm\eta\}$. Now, by Theorem 2.3 we find that $\eta < 2\sqrt{2}$. This yields that $m - m' > 4$, which contradicts $\Delta = 4$.

(c) $r = 1$, $s \geq 2$. Hence $m = 4s + 10$. Now, by Theorem 2.8 there is a vertex u of G such that either $R(G - u) = \{0^{t+1}, (\pm 2)^{s+1}\}$ or $R(G - u) = \{0^{t-1}, (\pm 2)^{s+1}, \pm\eta\}$, for some η, $0 < \eta \leq 1$. Let m' be the size of $G - u$. In the former, we have $m' = 4s + 4$ and so $m - m' = 6$, a contradiction. In the latter, we have $m' = \eta^2 + 4s + 4$. Thus $m - m' \geq 5$, a contradiction.

(d) $r, s \geq 2$, $t = 1$. It follows that there exists a vertex u of G such that either $R(G - u) = \{(\pm 1)^r, (\pm 2)^{s+1}\}$ or $R(G - u) = \{(\pm 1)^{r-1}, (\pm 2)^{s+1}, \pm\eta\}$, for some η, $0 \leq \eta \leq 1$. In the former, we have $m' = 4s + r + 4$ and so $m - m' = 5$, a contradiction. In the latter, we have $m' = \eta^2 + 4s + r + 3$. Hence $m - m' \geq 5$, a contradiction.

(e) $r, s, t \geq 2$. Therefore there is a vertex u of G such that $R(G - u) = \{0^{t-1}, (\pm 1)^r, (\pm 2)^{s+1}\}$, $R(G - u) = \{0^{t+1}, (\pm 1)^{r-1}, (\pm 2)^{s+1}\}$ or $R(G - u) = \{0^{t-1}, (\pm 1)^{r-1}, (\pm 2)^{s+1}, \pm\eta\}$, for some η, $0 < \eta < 1$. Let m' be the size of $G - u$. We distinguish three cases:

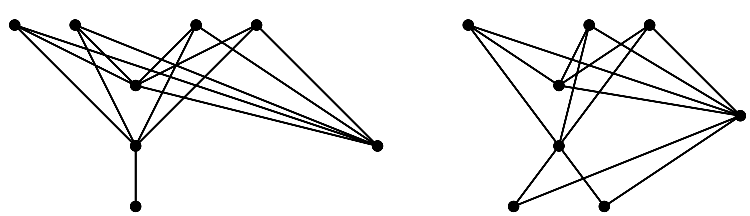
If $R(G - u) = \{0^{t-1}, (\pm 1)^r, (\pm 2)^{s+1}\}$, then $m' = r + 4s + 4$, and so $m - m' = 5$, a contradiction.

If $R(G - u) = \{0^{t+1}, (\pm 1)^{r-1}, (\pm 2)^{s+1}\}$, then $m' = r + 4s + 3$, and so $m - m' = 6$, a contradiction.

If $R(G - u) = \{0^{t-1}, (\pm 1)^{r-1}, (\pm 2)^{s+1}, \pm\eta\}$, then $m' = \eta^2 + 4s + r + 3$. Now, since $0 < \eta < 1$, $m - m' > 5$, a contradiction. This completes the proof. □

In [1], matching integral traceable graphs have been characterized. In the following theorem we determine all matching integral graphs whose longest path avoids only one vertex.

Theorem 3.4. *Let G be a connected graph on n vertices whose length of its longest path is $n - 1$. If G is matching integral, then it is one of the following graphs:*



Proof. We have $t(G) = n - 1$. Now, Theorem 2.5 implies that, the number of distinct matching zeros of G is at least $n - 1$ and multiplicity of each zero is at most two.

First, assume that G has n distinct matching zeros, i.e., all zeros are simple. Now, by Theorem 2.9, we see that G is traceable, a contradiction. Next, assume that G has $n - 1$ distinct matching zeros. It follows that $R(G) = \{0^2, \pm\theta_1, \dots, \pm\theta_k\}$, where $0 < \theta_1 < \cdots < \theta_k$ and for each i, $1 \leq i \leq k$,

θ_i is an integer. Note that, G is not a complete graph. We have $n = 2k+2$ and

$$\begin{aligned}\mu(G,x) &= x^2\prod_{i=1}^{k}(x^2-\theta_i^2)\\ &= x^{2k+2}-(\theta_1^2+\cdots+\theta_k^2)x^{2k}\\ &+\cdots+(-1)^k\theta_1^2\cdots\theta_k^2x^2.\end{aligned}$$

Hence

$$\begin{aligned}m=\sum_{i=1}^{k}\theta_i^2 \geq \sum_{i=1}^{k} i^2 &= \frac{k(k+1)(2k+1)}{6}\\ &= \frac{n}{12}\left(\frac{n}{2}-1\right)(n-1)\,.\end{aligned}$$

It follows that for $n \geq 14$, $m \geq \binom{n}{2}$, a contradiction. We consider three cases:

Case 1. $n \leq 8$. In this case, G is one of the graphs on eight vertices in Theorem 2.10.

Case 2. $n = 10$. Since $\Delta \leq 9$, Theorem 2.3 implies that matching zeros of G lie in the interval $[-5,5]$. As $m = \theta_1^2+\theta_2^2+\theta_3^2+\theta_4^2 < 45$, the positive matching zeros of G are either $1,2,3,4$ or $1,2,3,5$. In the first case, we have $\mu(G,x) = x^2(x^2-1)(x^2-4)(x^2-9)(x^2-16)$. In the second case, we have $\mu(G,x) = x^2(x^2-1)(x^2-4)(x^2-9)(x^2-25)$. However, a computer search shows that such graphs do not exist.

Case 3. $n = 12$. Since $\Delta \leq 11$, Theorem 2.3 implies that matching zeros of G lie in the interval $[-6,6]$. As $m = \theta_1^2+\theta_2^2+\theta_3^2+\theta_4^2+\theta_5^2 < 66$, we find that the positive matching zeros of G are $1,2,3,4,5$. Hence $\mu(G,x) = x^2(x^2-1)(x^2-4)(x^2-9)(x^2-16)(x^2-25)$. But, by a computer search we find that such graph does not exist. □

Acknowledgments

The author wish to thank Islamic Azad University, Science and Research Branch for the partial support.

References

[1] S. Akbari, P. Csikvári, A. Ghafari, S. Khalashi Ghezelahmad and M. Nahvi, Graphs with integer matching polynomial zeros, *Discrete Appl. Math.*, **224** (2017) 1–8.

[2] K. T. Balińska, D. Cvetković, Z. Radosavljević, S. K. Simić and D. Stevanović, A survey on integral graphs, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat.*, **13** (2002) 42–65.

[3] E. J. Farrell, An introduction to matching polynomials, *J. Combin. Theory Ser. B*, **27** no. 1 (1979) 75–86.

[4] E. Ghorbani, Graphs with few matching roots, *Graphs Combin.*, **29** no. 5 (2013) 1377–1389.

[5] C. D. Godsil, *Algebraic Combinatorics*, Chapman and Hall Mathematics Series, Chapman & Hall, New York, 1993.

[6] C. D. Godsil, Algebraic matching theory, *Electron. J. Combin.*, **2** (1995) 1–14.

[7] C. D. Godsil and I. Gutman, On the theory of the matching polynomial, *J. Graph Theory*, **5** no. 2 (1981) 137–144.

[8] I. Gutman, The matching polynomial, *MATCH Commun. Math. Comput. Chem.*, no. 6 (1979) 75–91.

[9] I. Gutman, Uniqueness of the matching polynomial, *MATCH Commun. Math. Comput. Chem.*, **55** no. 2 (2006) 351–358.

[10] F. Harary and A. J. Schwenk, Which graphs have integral spectra?, *Graphs and Combinatorics, Lecture Notes in Math.*, Springer-Verlag, Berlin,**406** (1974) 45–51.

[11] O. J. Heilmann and E. H. Lieb, Theory of monomer-dimer systems, *Comm. Math. Phys.*, **25** (1972) 190–232.

[12] S. Khalashi Ghezelahmad, On matching integral graphs, *Math. Sci.*, **13** (2019) 387–394.

[13] S. Khalashi Ghezelahmad, Matching integral graphs of small order, *JNRM*, **6** no. 26 (2020) 99–111.

[14] C. Y. Ku and W. Chen, An analogue of the Gallai-Edmond structure theorem for non-zero roots of the matching polynomial, *J. Combin. Theory Ser. B*, **100** no. 2 (2010) 119–127.

Somayeh Khalashi Ghezelahmad

Department of Mathematics and Computer Science, SR.C, Islamic Azad University, Tehran, Iran

`Email: ghezelahmad@iau.ac.ir`