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k-ODD MEAN LABELING OF PRISM

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ABSTRACT. A (p, q) graph G is said to have a k-odd mean labeling $(k \geq 1)$ if there exists an injection $f : V \to \{0, 1, 2, \dots, 2k + 2q - 3\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection from E to $\{2k - 1, 2k + 1, 2k + 3, \dots, 2k + 2q - 3\}$. A graph that admits k-odd mean labeling is called k-odd mean graph. In this paper, we investigate k-odd mean labeling of prism $C_m \times P_n$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [11]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Graph labeling was first introduced in the late 1960s. Many studies in graph labeling refer to Rosas research in 1967 [13]. Mean labeling of graphs was discussed in [14] and the concept of odd mean labeling was introduced in [12]. In [6], we introduced k-odd mean labeling and in [7] we extended k-odd mean labeling to (k, d)-odd mean labeling. In this paper, we investigate k-odd mean labeling of Prism $(C_m \times P_n)$.

2. Definitions

Definition 2.1 (k-odd mean labeling)**.** *A (p, q) graph G is said to have a k-odd mean labeling $(k \geq 1)$ if there exists a injection $f : V \to \{0, 1, 2, \dots, 2k + 2q - 3\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is an bijection from E to $\{2k - 1, 2k + 1, 2k + 3, \dots, 2k + 2q - 3\}$.*

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A graph that admits a k-odd mean labeling is called a k-odd mean graph.

Definition 2.2 ((k,d)-odd mean labeling). *A (p,q) graph G is said to have a (k,d)-odd mean labeling ($k \geq 1$ and $d \geq 1$) if there exists a injection $f : V \to \{0, 1, 2, \ldots, 2k-1+2(q-1)d\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is an bijection from E to $\{2k-1, 2k-1+2d, 2k-1+4d, \ldots, 2k-1+2(q-1)d\}$.*

A graph that admits a (k,d)-odd mean labeling is called a (k,d)-odd mean graph.

Remark 2.3. *1-odd mean labeling is an odd mean labeling.*

Remark 2.4. *The graphs C_3 and C_6 are not k-odd mean graphs.*

Definition 2.5. *The Cartesian product $G_1 \times G_2$ of two graphs G_1 and G_2 is the simple graph with $V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ iff either $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2, or u_1 is adjacent to u_2 in G_1 and $v_1 = v_2$.*

Remark 2.6. *For brevity, we use k-OML for k-odd mean labeling.*

3. Main result

Theorem 3.1. *$C_m \times P_n$ ($m \geq 4, n \geq 2$) is a k-odd mean graph for any k and $m \neq 6$.*

Proof.

$$V(C_m \times P_n) = \{v_{ij},\ 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \text{ and}$$

$$E(C_m \times P_n) = \{e_{ij},\ 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \cup \{e^{'}_{ij},\ 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\} \quad \text{(see Fig. 1).}$$

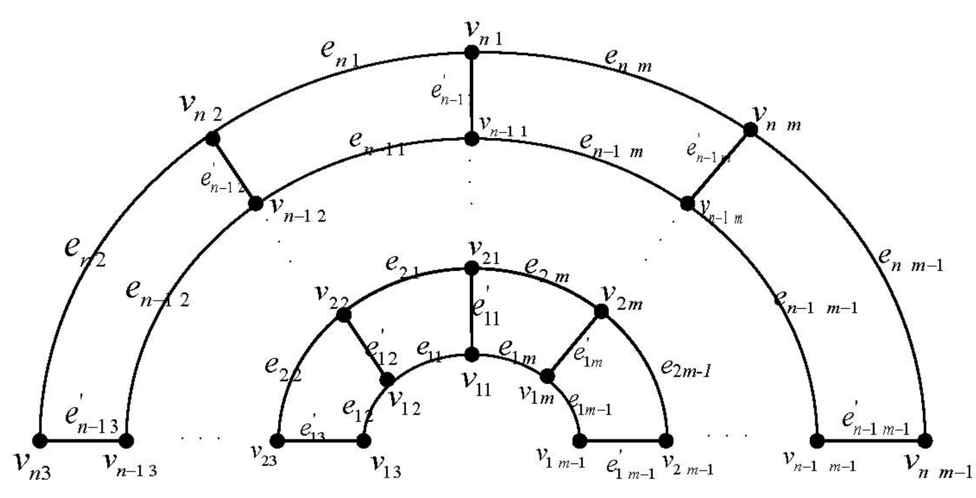

FIGURE 1. Ordinary labeling of $C_m \times P_n$

First we label the vertices of $C_m \times P_n$ as follows:

Define $f : V(C_m \times P_n) \to \{0, 1, 2, \ldots, 2k + 2q - 3\}$ by

Case (i) $m \equiv 0 \pmod 4$.

For $1 \le i \le n$ and i is odd,

for $1 \le j \le \frac{m}{2}$,

$$f(v_{ij}) = \begin{cases} 2k + 4m(i-1) + 4j - 6, & \text{if } j \text{ is odd,} \\ 2k + 4m(i-1) + 4j - 8, & \text{if } j \text{ is even} \end{cases}$$

for $\frac{m+2}{2} \le j \le m$,

$$f(v_{ij}) = \begin{cases} 2k + 4(mi - j) + 1, & \text{if } j \text{ is odd,} \\ 2k + 4(mi - j + 1), & \text{if } j \text{ is even.} \end{cases}$$

For $1 \le i \le n$ and i is even,

for $1 \le j \le \frac{m-2}{2}$,

$$f(v_{ij}) = \begin{cases} 2k + 4m(i-1) + 4(j-1), & \text{if } j \text{ is odd,} \\ 2k + 4m(i-1) + 4(j-1) - 2, & \text{if } j \text{ is even} \end{cases}$$

for $\frac{m}{2} \le j \le m$,

$$f(v_{ij}) = \begin{cases} 2k + 4(mi - j), & \text{if } j \text{ is odd,} \\ 2k + 4(mi - j) - 3, & \text{if } j \text{ is even.} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le n$ and i is odd,

$$f^*(e_{ij}) = 2k + 4m(i-1) + 4j - 5, \ 1 \le j \le \frac{m}{2},$$
$$f^*\left(e_{i\frac{m+2}{2}}\right) = 2k + 2m + 4m(i-1) - 3,$$
$$f^*(e_{ij}) = 2k + 4mi - 4j + 1, \ \frac{m+4}{2} \le j \le m.$$

For $1 \le i \le n$ and i is even,

$$f^*(e_{ij}) = \begin{cases} 2k + 4m(i-1) + 4(j-1) + 3, & 1 \le j \le \frac{m-2}{2}, \\ 2k + 4mi - 4j - 3, & \frac{m}{2} \le j \le m-1. \end{cases}$$
$$f^*(e_{im}) = 2k + 4m(i-1) - 1.$$

For $1 \le i \le n-1$,

$$f^*(e'_{ij}) = \begin{cases} 2k + 2m(2i-1) + 4j - 5, & 1 \le j \le \frac{m}{2}, \\ 2k + 2m(2i-1) + 4j - 3, & \frac{m+2}{2} \le j \le m-1. \end{cases}$$

3-OML of $C_8 \times P_3$ is shown in Fig. 2.

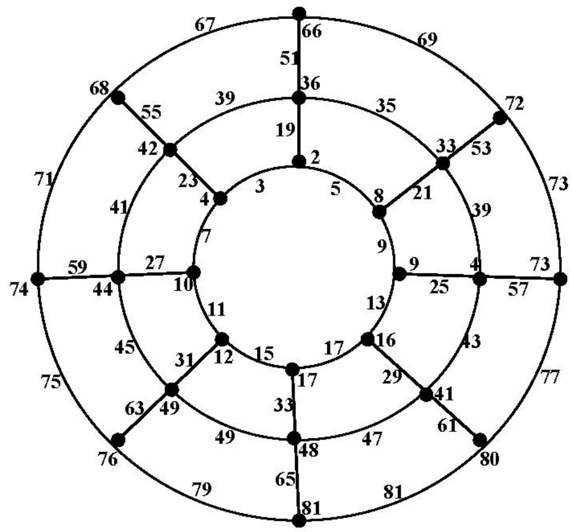

FIGURE 2. 3-OML of $C_8 \times P_3$.

Case (ii) $m \equiv 1,\ 3 \pmod 4$.

The vertex labels are:

For $1 \leq i \leq n$ and i is odd,

for $1 \leq j \leq \frac{m-3}{2}$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-2),$$

$$f\left(v_{i\frac{m-1}{2}}\right) = 2k + m(4i-3) - 6,$$

$$f\left(v_{i\frac{m+1}{2}}\right) = 2k + m(4i-3) - 2$$

for $\frac{m+3}{2} \leq j \leq m-1$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-1),$$

$$f(v_{im}) = 2k + 2m(2i-1) - 3.$$

For $1 \leq i \leq n$ and i is even,

for $1 \leq j \leq \frac{m-5}{2}$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-1),$$

$$f\left(v_{i\frac{m-1}{2}}\right) = 2k + m(4i-3) - 2$$

for $\frac{m+1}{2} \leq j \leq m-2$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2j,$$

$$f(v_{im-1}) = 2k + 4m(i-1) + 2n - 3,$$

$$f(v_{im}) = 2k + 4m(i-1) - 2.$$

Then the induced edge labels are:
For $1 \leq i \leq n$ and i is odd,

$$f^*(e_{ij}) = \begin{cases} 2k + 4m(i-1) + 2j - 3, 1 \leq j \leq \frac{m-1}{2}, \\ 2k + 4m(i-1) + 2j - 1, \frac{m+1}{2} \leq j \leq m-1. \end{cases}$$

$$f^*(e_{im}) = 2k + m(4i-3) - 2.$$

For $1 \leq i \leq n$ and i is even,
for $1 \leq j \leq \frac{m-3}{2}$,

$$f^*(e_{ij}) = 2k + 4m(i-1) + 2j - 1$$

for $\frac{m-1}{2} \leq j \leq m-2$,

$$f^*(e_{ij}) = 2k + 4m(i-1) + 2j + 1,$$

$$f^*(e_{im-1}) = 2k + m(4i-3) - 2,$$

$$f^*(e_{im}) = 2k + 4m(i-1) - 1.$$

For $1 \leq i \leq n-1$ and $1 \leq j \leq \frac{m-1}{2}$,

$$f^*(e'_{ij}) = 2k + 2m(2i-1) + 2j - 3$$

for $\frac{m+1}{2} \leq j \leq m-1$,

$$f^*(e'_{ij}) = 2k + 2m(2i-1) + 2j - 1,$$

$$f^*(e'_{im}) = 2k + m(4i-1) - 2.$$

4-OML of $C_5 \times P_5$ is shown Fig. 3.

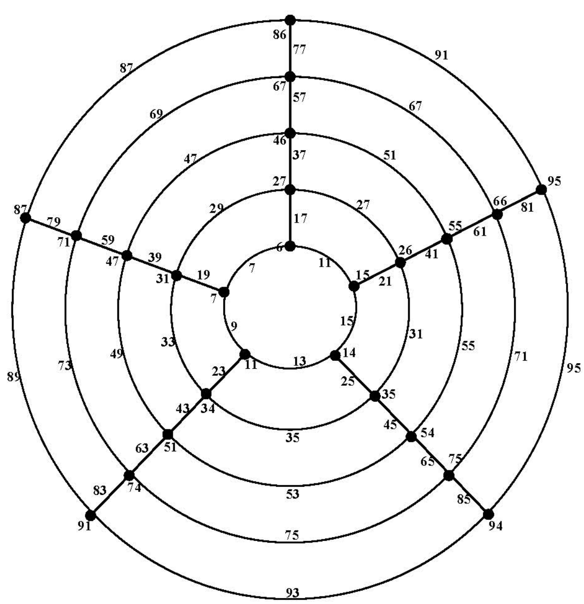

FIGURE 3. 4-OML of $C_5 \times P_5$.

5-OML of $C_{11} \times P_3$ is shown in Fig. 4.

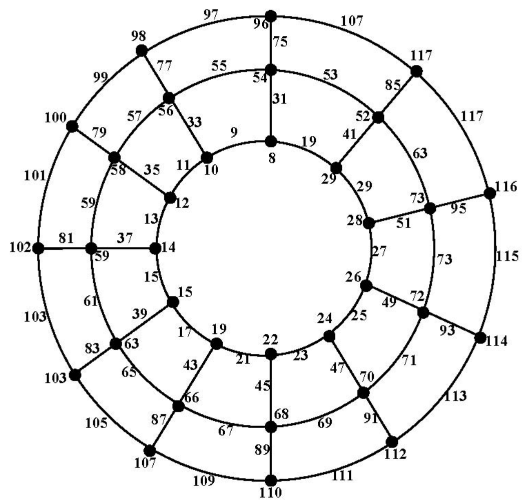

FIGURE 4. 5-OML of $C_{11} \times P_3$.

Case (iii) $m \equiv 2 \pmod 4$ and $m > 6$.
The vertex labels are:
For $1 \leq i \leq n$ and i is odd,
for $1 \leq j \leq \frac{m-4}{2}$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-2),$$
$$f\left(v_{i\frac{m-2}{2}}\right) = 2k + m(4i-3) - 7,$$
$$f\left(v_{i\frac{m}{2}}\right) = 2k + m(4i-3) - 3$$

for $\frac{m+2}{2} \leq j \leq m-3$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-1),$$
$$f(v_{im-2}) = 2k + 2m(2i-1) - 7,$$
$$f(v_{im-1}) = 2k + 2n(2i-1) - 3,$$
$$f(v_{im}) = 2k + 2m(2i-1) - 4.$$

For $1 \leq i \leq n$ and i is even,
for $1 \leq j \leq \frac{m-6}{2}$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2(j-1),$$
$$f\left(v_{i\frac{m-4}{2}}\right) = 2k + m(4i-3) - 7,$$
$$f\left(v_{i\frac{m-2}{2}}\right) = 2k + m(4i-3) - 3$$

for $\frac{m}{2} \leq j \leq m-4$,

$$f(v_{ij}) = 2k + 4m(i-1) + 2j,$$
$$f(v_{im-3}) = 2k + 2m(2i-1) - 7,$$
$$f(v_{im-2}) = 2k + 2m(2i-1) - 3,$$
$$f(v_{im-1}) = 2k + 2m(2i-1) - 4,$$
$$f(v_{im}) = 2k + 4m(i-1) - 2.$$

Then the induced edge labels are:
For $1 \leq i \leq n$ and i is odd,

$$f^*(e_{ij}) = \begin{cases} 2k + 4m(i-1) + 2j - 3, 1 \leq j \leq \frac{m-2}{2}, \\ 2k + 4m(i-1) + 2j - 1, \frac{m}{2} \leq j \leq m-1, \end{cases}$$
$$f^*(e_{im}) = 2k + m(4i-3) - 3.$$

For $1 \leq i \leq n$ and i is even,
for $1 \leq j \leq \frac{m-4}{2}$,

$$f^*(e_{ij}) = 2k + 4m(i-1) + 2j - 1$$

for $\frac{m-2}{2} \leq j \leq m-2$,

$$f^*(e_{ij}) = 2k + 4m(i-1) + 2j + 1,$$
$$f^*(e_{im-1}) = 2k + m(4i-3) - 3,$$
$$f^*(e_{im}) = 2k + 4m(i-1) - 1.$$

For $1 \leq i \leq n-1$ and $1 \leq j \leq \frac{m-2}{2}$,

$$f^*(e'_{ij}) = 2k + 2m(2i-1) + 2j - 3.$$

For $\frac{m}{2} \leq j \leq m-1$,

$$f^*(e'_{ij}) = 2k + 2m(2i-1) + 2j - 1,$$
$$f^*(e'_{im}) = 2k + m(4i-1) - 3.$$

2-OML of $C_{14} \times P_3$ is shown in

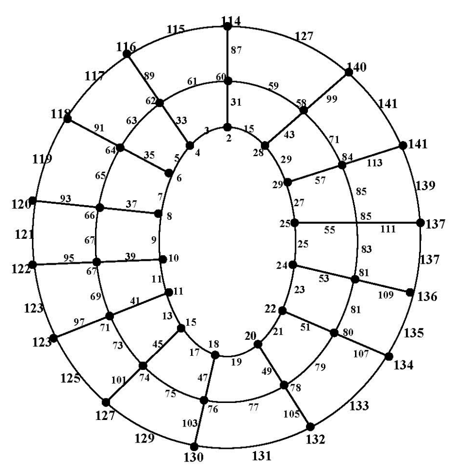

FIGURE 5. 2-OML of $C_{14} \times P_3$.

Therefore, $f^*(E(C_m \times P_n) = \{2k-1, 2k+1, 2k+3, \ldots, 2k+2q-3\}$.

So, f is a k-odd mean labeling and hence, $C_m \times P_n$ is k-odd mean graph for any k when $m \geq 4$, $n \geq 2$ and $m \neq 6$.

□

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