www.combinatorics.ir

Transactions on Combinatorics
ISSN (print): 2251-8657, ISSN (on-line): 2251-8665
Vol. 4 No. 3 (2015), pp. 1-9.

www.ui.ac.ir

THE HYPER EDGE-WIENER INDEX OF CORONA PRODUCT OF GRAPHS

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Communicated by Ivan Gutman

ABSTRACT. Let G be a simple connected graph. The edge-Wiener index $W_e(G)$ is the sum of all distances between edges in G, whereas the hyper edge-Wiener index $WW_e(G)$ is defined as $WW_e(G) = \frac{1}{2}W_e(G) + \frac{1}{2}W_e^2(G)$, where $W_e^2(G) = \sum_{\{f,g\}\subseteq E(G)} d_e^2(f,g)$. In this paper, we present explicit formula for the hyper edge-Wiener index of corona product of two graphs. Also, we use it to determine the hyper edge-Wiener index of some chemical graphs.

1. Introduction

A topological index is a real number derived from the structure of a graph. Chemical graph theory is a branch of mathematical chemistry that is mostly concerned with finding topological indices of chemical graphs. The Wiener index is the first distance based topological index that was introduced by H. Wiener in 1947 [19] in a paper concerned with boiling points of alkanes. This index is defined as the sum of all distances between different vertices of a molecular graph. In other words, if G is a simple connected graph with vertex set $V(G)$ and edge set $E(G)$, respectively, then the Wiener index is defined as:

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d(u,v)$$

where $d(u,v)$ or $d(u,v|G)$ is the length of any shortest path in G connecting u and v.

This index has many applications. We refer the reader to [4, 5, 15] for more facts concerning its mathematical properties and chemical applications. Nowadays, there are many topological indices

MSC(2010): Primary: 05C12; Secondary: 05C90, 05C76.

Keywords: Distance, Topological index, Hyper edge-Wiener index, Corona product.

Received: 17 August 2014, Accepted: 23 August 2014.

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and most of them have applications in chemistry, biochemistry, computer science and nanotechnology [18].

Randić introduced a modification of the Wiener index for trees, and it is known as the hyper Wiener index [16]. Klein et al., see [12], generalized this extension to cyclic structures as:

$$WW(G) = \frac{1}{2}W(G) + \frac{1}{2}W^2(G)$$

where $W^2(G) = \sum_{\{u,v\}\in V(G)} d^2(u,v)$.

The above indices are based on the distances between vertices of graph G. If we consider the line graph $L(G)$ rather than G, then we can introduce some new indices.

Using this remark, recently another index, called the edge-Wiener index, is defined in [9] as fallow:

$$W_e(G) = \sum_{\{f,g\}\subseteq E(G)} d_e(f,g)$$

where $d_e(f,g)$ or $d_e(f,g|G)$ is the distance between the vertices f and g in the line graph of G. This distance is equal to $\min\{d(x,u), d(x,v), d(y,u), d(y,v)\} + 1$.

In other words, the edge-Wiener index of graph G is the Wiener index of its line graph $L(G)$.

Also similarly, the hyper edge-Wiener index is defined as fallow [10]:

$$WW_e(G) = \frac{1}{2}W_e(G) + \frac{1}{2}W_e^2(G)$$

where $W_e^2(G) = \sum_{\{f,g\}\subseteq E(G)} d_e^2(f,g)$.

The reader can see [2, 10, 11] for topics and researches related to these indices. Recently in [17], we introduced a generalization of these indices and the hyper edge-Wiener index of a graph product, the join of graphs, is computed.

In this paper, we determine the hyper edge-Wiener index of corona product. First of all, we need some essential concepts and definitions.

For two graphs G_1 and G_2, the *corona* product, denoted by $G_1 \circ G_2$, is obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and joining all vertices of the i-th copy of G_2 to the i-th vertex of G_1 for $i = 1, 2, \cdots, |V(G_1)|$. Note that this product is not commutative.

By specializing the components in the corona product, some classes of graphs can also be obtained. For example, the *m-thorny* graph of a given graph G is obtained as $G \circ \overline{K}_m$, where $\overline{K}_m$ denotes the empty graph on m edges [13]. Also, the graphs $K_1 \circ G$ and $K_2 \circ G$ are called the *suspension* and *bottleneck* graphs for a given graph G, respectively. All these graphs are given in chemical literature, on other words, the corona product is useful in chemical graph theory. For more information concerning applications of the corona product in molecular graph theory see [14].

Let $N(u)$ denote the neighborhood of a vertex u in G, i.e., the set of all vertices of G adjacent with u. In [1], we have the notation $N(G)$ that is defined as follows:

$$N(G) = \sum_{uv\in E(G)} \sum_{z\in V(G)\backslash(N(u)\cup N(v))} |N(z)\backslash(N(u)\cup N(v))|.$$

Each pair of distinct edges in G is counted exactly four times by $N(G)$, hence the number of pairs of distinct edges of G is equal to $\frac{1}{4}N(G)$. It is verified that $N(P_n) = N(C_n) = 0$ for $n < 5$ and that $\frac{1}{4}N(P_n) = \frac{1}{2}(n-4)(n-3)$ and $\frac{1}{4}N(C_n) = \frac{1}{2}n(n-5)$ for $n \geq 5$.

Another concept that will help us formulate our results in a more compact way, is the Zagreb indices for a graph G. They are defined as follows [6]:

$$M_1(G) = \sum_{u \in V(G)} \delta^2(u), \qquad M_2(G) = \sum_{e=uv \in E(G)} \delta(u)\,\delta(v),$$

where $\delta(u)$ denotes the degree of vertex u.

In the next section, we aim to present our main result. Our formulas are given in terms of the first Zagreb index and $N(G)$.

We note that all graphs considered in this paper are finite and simple. For the concepts not defined here, we refer the reader to any standard graph theory book such as [8].

2. **Main Results**

Throughout this section, let G_1 and G_2 be two finite simple connected graphs and n_i and e_i denote the numbers of vertices and edges of G_i, respectively, where $i \in \{1, 2\}$. In this section, we compute the hyper edge-Wiener index of corona product of G_1 and G_2. Before we come to this, we begin by defining two quantities introduced in [1]. Let x be a vertex of G and $f = uv$ be an edge of G. Then $D(x|f) = \min\{d(x,u), d(x,v)\}$. $D(x|G)$ is obtained by summing such contributions over all edges of G, $D(x|G) = \sum_{f \in E(G)} D(x|f)$, and $W_{ve}(G)$ is obtained by summing $D(x|G)$ over all vertices $x \in V(G)$,

$$W_{ve}(G) = \sum_{x \in V(G)} \sum_{f \in E(G)} D(x|f) = \sum_{x \in V(G)} D(x|G).$$

Similarly, in this paper we define $D^2(x|f) = \min\{d^2(x,u), d^2(x,v)\}$ and so we have

$$D^2(x|G) = \sum_{f \in E(G)} D^2(x|f) \text{ and } W^2_{ve}(G) = \sum_{x \in V(G)} D^2(x|G).$$

Also, we define $WW_{ve}(G) = \frac{1}{2}W_{ve}(G) + \frac{1}{2}W^2_{ve}(G)$.

To achieve our goal, first we partition the edges of $G_1 \circ G_2$ into three sets:

$$E_1 = \{e \in E(G_1 \circ G_2) |\ e \in E(G_1)\}$$

$$E_2 = \{e \in E(G_1 \circ G_2) |\ e \in \text{copies of } G_2\}$$

$$E_3 = \{e \in E(G_1 \circ G_2) |\ e = xu; x \in V(G_1) \text{ and } u \in V(G_2)\}$$

The corona product $G_1 \circ G_2$ has $e_1 + n_1e_2 + n_1n_2$ edges. The copy of G_2 related to the vertex $x \in V(G_1)$ is denoted by $G_{2,x}$ and the edge set of $G_{2,x}$ by $E_{2,x}$, so $E_2 = \bigcup_{x \in V(G_1)} E_{2,x}$. Also, set $E_{3,x} = \{e | e = xu, u \in V(G_{2,x})\}$, and so $E_3 = \bigcup_{x \in V(G_1)} E_{3,x}$, see Fig. 1.

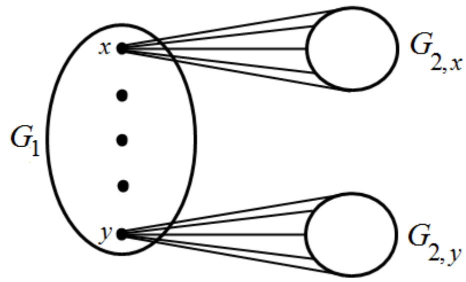

FIGURE 1. The corona product of two graphs G_1 and G_2.

We need the following lemma:

Lemma 2.1. [1] *Let G_1 and G_2 be two connected graphs. Then*

1) *If $\{f, g\} \subseteq E_1$, then $d_e(f, g|G_1 \circ G_2) = d_e(f, g|G_1)$*
2) *If $\{f, g\} \subseteq E_2$, $f \in E_{2,x}$ and $g \in E_{2,y}$, then*
$$d_e(f, g|G_1 \circ G_2) = \begin{cases} 1,\ 2 \text{ or } 3 & \text{if } x = y \\ d(x, y|G_1) + 3 & \text{if } x \neq y. \end{cases}$$
3) *If $\{f, g\} \subseteq E_3$, $f \in E_{3,x}$ and $g \in E_{3,y}$, then*
$$d_e(f, g|G_1 \circ G_2) = \begin{cases} 1 & \text{if } x = y \\ d(x, y|G_1) + 1 & \text{if } x \neq y. \end{cases}$$
4) *Let $f \in E_1$ and $g \in E_2$. If $g \in E_{2,x}$, then $d_e(f, g|G_1 \circ G_2) = 2 + D(x|f)$ in G_1.*
5) *Let $f \in E_1$ and $g \in E_3$. If $g \in E_{3,x}$, then $d_e(f, g|G_1 \circ G_2) = 1 + D(x|f)$ in G_1.*
6) *If $f \in E_2$ and $g \in E_3$ such that $f \in E_{2,x}$ and $g \in E_{3,y}$, then*
$$d_e(f, g|G_1 \circ G_2) = \begin{cases} 1 \text{ or } 2 & \text{if } x = y \\ d(x, y|G_1) + 2 & \text{if } x \neq y. \end{cases}$$

The next theorem is also necessary for computing the hyper edge-Wiener index of corona product.

Theorem 2.2. [1] *Let G_1 and G_2 be two connected graphs. Then*

$$\begin{aligned} W_e(G_1 \circ G_2) =& W_e(G_1) + (e_2 + n_2)^2 W(G_1) - \frac{n_1}{2} M_1(G_2) + \frac{n_1}{4} N(G_2) \\ &+ e_2^2 \left[3\binom{n_1}{2} + n_1\right] + n_1 \binom{n_2}{2} + n_2^2 \binom{n_1}{2} + n_1 e_1 (2e_2 + n_2) \\ &+ 2n_1 e_2 (n_2 - 1) + 2n_1 n_2 e_2 (n_1 - 1) + (e_2 + n_2) W_{ve}(G_1). \end{aligned}$$

Now, we have to determine $W_e^2(G_1 \circ G_2)$. So we provide the following theorem:

Theorem 2.3. *Let G_1 and G_2 be two connected graphs. Then*

$$\begin{aligned}W_e^2(G_1\circ G_2) =& W_e^2(G_1) + (6e_2^2+2n_2^2+8n_2e_2)W(G_1) + (e_2+n_2)^2W^2(G_1)\\&-\frac{3n_1}{2}M_1(G_2)+\frac{5n_1}{4}N(G_2)+e_2^2\left[9\binom{n_1}{2}+2n_1\right]+n_1\binom{n_2}{2}+n_1e_2\\&+n_2^2\binom{n_1}{2}+n_1e_1(4e_2+n_2)+2n_1e_2(2n_2-3)+4n_1n_2e_2(n_1-1)\\&+(4e_2+2n_2)W_{ve}(G_1)+(e_2+n_2)W_{ve}^2(G_1).\end{aligned}$$

Proof. Let $G=G_1\circ G_2$. Using 6 parts of Lemma 2.1, there are some cases:

Case 1) $\{f,g\}\subseteq E_1$.

According to part 1 of Lemma 2.1, we have

$$W_{e1}^2=\sum_{\{f,g\}\subseteq E_1} d_e^2(f,g|G)=\sum_{\{f,g\}\subseteq E_1} d_e^2(f,g|G_1)=W_e^2(G_1).$$

Case 2) $\{f,g\}\subseteq E_2$, $f\in E_{2,x}$ and $g\in E_{2,y}$.

Using part 2 of Lemma 2.1, If $\{f,g\}\in E_{2,x}$, means $x=y$, then no pair of edges from $E_{2,x}$ is at a distance greater than 3. Hence, we partition $E_{2,x}$ into three sets, $E'_{2,x}$, $E''_{2,x}$ and $E'''_{2,x}$, made of the pairs of edges at distance 1, 2 and 3 in G, respectively. The total contribution of pairs from $E_{2,x}$ to W_e^2 of $G_1\circ G_2$ is given by $|E'_{2,x}|+2^2|E''_{2,x}|+3^2|E'''_{2,x}|$. In section 1, We have mentioned that $|E'''_{2,x}|=\frac{1}{4}N(G_2)$. Further,

$$|E'_{2,x}|=\sum_{u\in V(G_2)}\binom{\delta(u)}{2}=\frac{1}{2}M_1(G_2)-e_2.$$

The number of pairs $\{f,g\}$ in $E_{2,x}$ is equal to $\binom{e_2}{2}$ and therefore the total contribution of $E_{2,x}$ is given by

$$\begin{aligned}\sum_{\{f,g\}\subseteq E_{2,x}} d_e^2(f,g|G) &= \left(\frac{1}{2}M_1(G_2)-e_2\right)+2^2\left(\binom{e_2}{2}-\left(\frac{1}{2}M_1(G_2)-e_2\right)-\left(\frac{1}{4}N(G_2)\right)\right)\\&+3^2\left(\frac{1}{4}N(G_2)\right)=2e_2^2+e_2-\frac{3}{2}M_1(G_2)+\frac{5}{4}N(G_2).\end{aligned}$$

Now we have

$$\begin{aligned}W_{e2}^2 &= \sum_{\{f,g\}\subseteq E_2} d_e^2(f,g|G)\\&=\sum_{x\in V(G_1)}\sum_{\{f,g\}\subseteq E_{2,x}} d_e^2(f,g|G)+\sum_{\{x,y\}\subseteq V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{2,y}} d_e^2(f,g|G)\\&=n_1\left(2e_2^2+e_2-\frac{3}{2}M_1(G_2)+\frac{5}{4}N(G_2)\right)+\sum_{\{x,y\}\subseteq V(G_1)}(3+d(x,y|G_1))^2e_2^2\\&=n_1\left(2e_2^2+e_2-\frac{3}{2}M_1(G_2)+\frac{5}{4}N(G_2)\right)+e_2^2\left(9\binom{n_1}{2}+6W(G_1)+W^2(G_1)\right).\end{aligned}$$

Case 3) According to part 3 of Lemma 2.1, we have

$$\begin{aligned}
W_{e3}^2 &= \sum_{\{f,g\}\subseteq E_3} d_e^2(f,g|G) \\
&= \sum_{x\in V(G_1)} \sum_{\{f,g\}\subseteq E_{3,x}} d_e^2(f,g|G) + \sum_{\{x,y\}\subseteq V(G_1)} \sum_{f\in E_{3,x}} \sum_{g\in E_{3,y}} d_e^2(f,g|G) \\
&= \sum_{x\in V(G_1)} \sum_{\{f,g\}\subseteq E_{3,x}} 1 + \sum_{\{x,y\}\subseteq V(G_1)} \sum_{f\in E_{3,x}} \sum_{g\in E_{3,y}} (d(x,y|G_1)+1)^2 \\
&= \sum_{x\in V(G_1)} \frac{1}{2} n_2(n_2-1) + \sum_{\{x,y\}\subseteq V(G_1)} n_2^2\ (d(x,y|G_1)+1)^2 \\
&= \sum_{x\in V(G_1)} \frac{1}{2} n_2(n_2-1) + n_2^2 \sum_{\{x,y\}\subseteq V(G_1)} (1+2d(x,y|G_1)+d^2(x,y|G_1)) \\
&= \frac{1}{2} n_1 n_2(n_2-1) + \frac{1}{2} n_2^2 n_1(n_1-1) + 2n_2^2\ W(G_1) + n_2^2\ W^2(G_1).
\end{aligned}$$

Case 4) According to part 4 of Lemma 2.1, we have

$$\begin{aligned}
W_{e4}^2 &= \sum_{\{(f,g)\}\subseteq E_1\times E_2} d_e^2(f,g|G) \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{2,x}} \sum_{f\in E_1} d_e^2(f,g|G) = \sum_{x\in E_1} \sum_{g\in E_{2,x}} \sum_{f\in E_1} (2+D(x|f))^2 \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{2,x}} \sum_{f\in E_1} \left(4+4D(x|f)+D^2(x|f)\right) \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{2,x}} \left(4e_1+4D(x|G_1)+D^2(x|G_1))\right) \\
&= \sum_{x\in V(G_1)} \left(4e_1+4D(x|G_1)+D^2(x|G_1)\right) e_2 = e_2\left(4n_1e_1+4W_{ve}(G_1)+W_{ve}^2(G_1)\right).
\end{aligned}$$

Case 5) According to part 5 of Lemma 2.1, we have

$$\begin{aligned}
W_{e5}^2 &= \sum_{\{(f,g)\}\subseteq E_1\times E_3} d_e^2(f,g|G) \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{3,x}} \sum_{f\in E_1} d_e^2(f,g|G) = \sum_{x\in E_1} \sum_{g\in E_{3,x}} \sum_{f\in E_1} (1+D(x|f))^2 \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{3,x}} \sum_{f\in E_1} \left(1+2D(x|f)+D^2(x|f)\right) \\
&= \sum_{x\in V(G_1)} \sum_{g\in E_{3,x}} \left(e_1+2D(x|G_1)+D^2(x|G_1))\right) \\
&= \sum_{x\in V(G_1)} \left(e_1+2D(x|G_1)+D^2(x|G_1)\right) n_2 = n_2\left(n_1e_1+2W_{ve}(G_1)+W_{ve}^2(G_1)\right).
\end{aligned}$$

Case 6) According to part 6 of Lemma 2.1, we have

$$\begin{aligned} W_{e6}^2 =& \sum_{\{(f,g)\}\subseteq E_2\times E_3} d_e^2(f,g|G) \\ =& \sum_{x\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,x}} d_e^2(f,g|G) + \sum_{x\neq y\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,y}} d_e^2(f,g|G) \\ =& \sum_{x\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,x}} d_e^2(f,g|G) + \sum_{x\neq y\in V(G_1)} (d(x,y|G_1)+2)^2 n_2e_2 \\ =& \sum_{x\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,x}} d_e^2(f,g|G) + \sum_{x\neq y\in V(G_1)} (4+4d(x,y|G_1)+d^2(x,y|G_1))n_2e_2 \\ =& \sum_{x\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,x}} d_e^2(f,g|G) + [4n_1(n_1-1)+8W(G_1)+2W^2(G_1)]n_2e_2 \end{aligned}$$

However, if $f\in E_{2,x}, g\in E_{3,x}$, then $d_e(f,g|G)=1$ or 2. The edge f is adjacent to two edges of $E_{3,x}$ and its distance to other edges is 2. So

$$\sum_{x\in V(G_1)}\sum_{f\in E_{2,x}}\sum_{g\in E_{3,x}} d_e^2(f,g|G) = \sum_{x\in V(G_1)}\sum_{f\in E_{2,x}} (2+4(n_2-2)) = 2(2n_2-3)e_2n_1$$

Hence we have

$$W_{e6}^2 = 2(2n_2-3)e_2n_1 + [2n_1(n_1-1)+8W(G_1)+W^2(G_1)]n_2e_2$$

Now by adding all the values of the above cases and simplifying the resulting expression, we deduce the desired result. □

Corollary 2.4. *Let G_1 and G_2 be two connected graphs. Then*

$$\begin{aligned} WW_e(G_1\circ G_2) =& WW_e(G_1) + (e_2+n_2)^2WW(G_1) + (e_2+n_2)(3e_2+n_2)W(G_1) \\ & - n_1M_1(G_2) + \frac{3n_1}{4}N(G_2) + e_2^2\left[6\binom{n_1}{2}+\frac{3}{2}n_1\right] + n_1\binom{n_2}{2} + n_2^2\binom{n_1}{2} \\ & + \frac{1}{2}n_1e_2 + n_1e_1(3e_2+n_2) + n_1e_2(3n_2-4) + 6\binom{n_1}{2}n_2e_2 \\ & + (e_2+n_2)WW_{ve}(G_1) + (2e_2+n_2)W_{ve}(G_1). \end{aligned}$$

Note that it is possible to apply this corollary to the cases of $G_1\circ G_2$ with disconnected G_2.

In the following, let G be a graph of order n and e be the number of its edges.

Corollary 2.5. *For the suspension graph $K_1\circ G$,*

$$WW_e(K_1\circ G) = 3\binom{n+e}{2} - 2\binom{n}{2} - 2e - M_1(G) + \frac{3}{4}N(G).$$

Now the formulas for the *wheel graph* $W_n = K_1\circ C_n$ and for the *fan graph* $K_1\circ P_n$ computed as follows:

Corollary 2.6. *For $n\geq 5$,*

$$WW_e(K_1\circ C_n) = n(13n-31)/2,$$
$$WW_e(K_1\circ P_n) = (13n^2-49n+58)/2.$$

Corollary 2.7. *For the bottleneck graph $K_2 \circ G$,*

$$WW_e(K_2 \circ G) = 8\binom{n+e}{2} - 10\binom{n}{2} + 6\binom{e}{2} + 5(e+n)^2 + 7e^2 - 2M_1(G) + \frac{3}{2}N(G).$$

Corollary 2.8. *For the m-thorny graph $G \circ \overline{K}_m$,*

$$WW_e(G \circ \overline{K}_m) = WW_e(G) + m^2\left[WW(G) + W(G) + \binom{n}{2}\right] + n\binom{m}{2} + m\left[ne + WW_{ve}(G) + W_{ve}(G)\right].$$

In the following table, we compute requested quantities for two graphs C_n and P_n.

	P_n	C_n (n odd)	C_n (n even)
W	$\binom{n+1}{3}$	$\frac{1}{4}\binom{n}{2}(n+1)$	$\frac{n^3}{8}$
WW	$\frac{1}{3}\binom{n}{2}\binom{n+2}{2}$	$\frac{1}{24}\binom{n}{2}(n+1)(n+3)$	$\frac{1}{24}\binom{n+1}{2}(n)(n+2)$
WW_e	$\frac{1}{3}\binom{n-1}{2}\binom{n+1}{2}$	$\frac{1}{24}\binom{n}{2}(n+1)(n+3)$	$\frac{1}{24}\binom{n+1}{2}(n)(n+2)$
W_{ve}	$2\binom{n}{3}$	$\frac{1}{2}\binom{n}{2}(n-1)$	$\frac{n^2(n-2)}{4}$
W_{ve}^2	$\binom{n}{3}(n-1)$	$\frac{1}{2}\binom{n}{2}(n-1) + \frac{1}{2}\binom{n}{3}(n-3)$	$\frac{1}{2}\binom{n}{3}n$
WW_{ve}	$\frac{1}{2}\binom{n}{3}(n+1)$	$\frac{1}{2}\binom{n}{2}(n-1) + \frac{1}{4}\binom{n}{3}(n-3)$	$\frac{n^2(n-2)}{8} + \frac{1}{4}\binom{n}{3}n$

Using Corollary 2.8 and the above table, by replacing the needed values, we can compute formulas for the *m-thorny cycle*, $C_n \circ \overline{K}_m$, and the *m-thorny path*, $P_n \circ \overline{K}_m$.

Finally, we mention that there are still some classes of chemically interesting graphs not covered here. In this paper and also in [17], the hyper edge-Wiener index of two graph products is studied separately. However, we can construct another new chemical graphs using other graph operations such as Cartesian product, the vertex-sum and edge-sum graphs. Therefore investigate about the hyper edge-Wiener index under such these graph products will be interesting.

Acknowledgments

The authors wish to thank the referee for helpful comments and suggestions. This work was partially supported by Center of Excellence of Algebraic Hyperstructures and its Applications of Tarbiat Modares University (CEAHA).

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